Efficient NIZKs from LWE via Polynomial Reconstruction and "MPC in the Head"

Riddhi Ghosal
UCLA

Paul Lou
UCLA

Amit Sahai
UCLA

NIZKs for all of NP from LWE [CCH+19, PS19]

Prior to our work, all known NIZK arguments for NP from LWE considered instantiating the Fiat-Shamir paradigm on a ***parallel repetition*** of a public-coin honest-verifier zero-knowledge interactive proof:

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$x \in L \Rightarrow x' \in \mathrm{HAM}$

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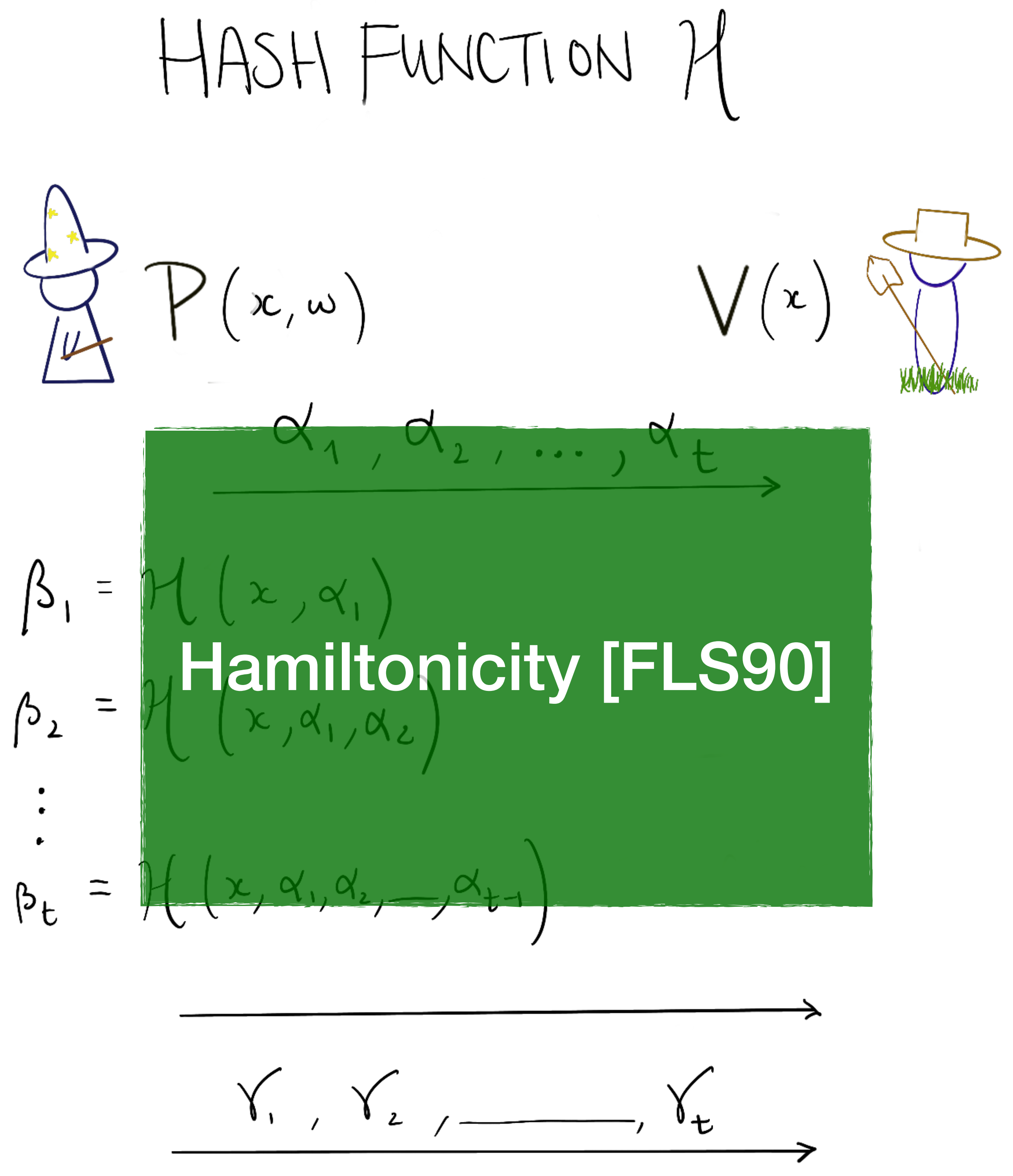

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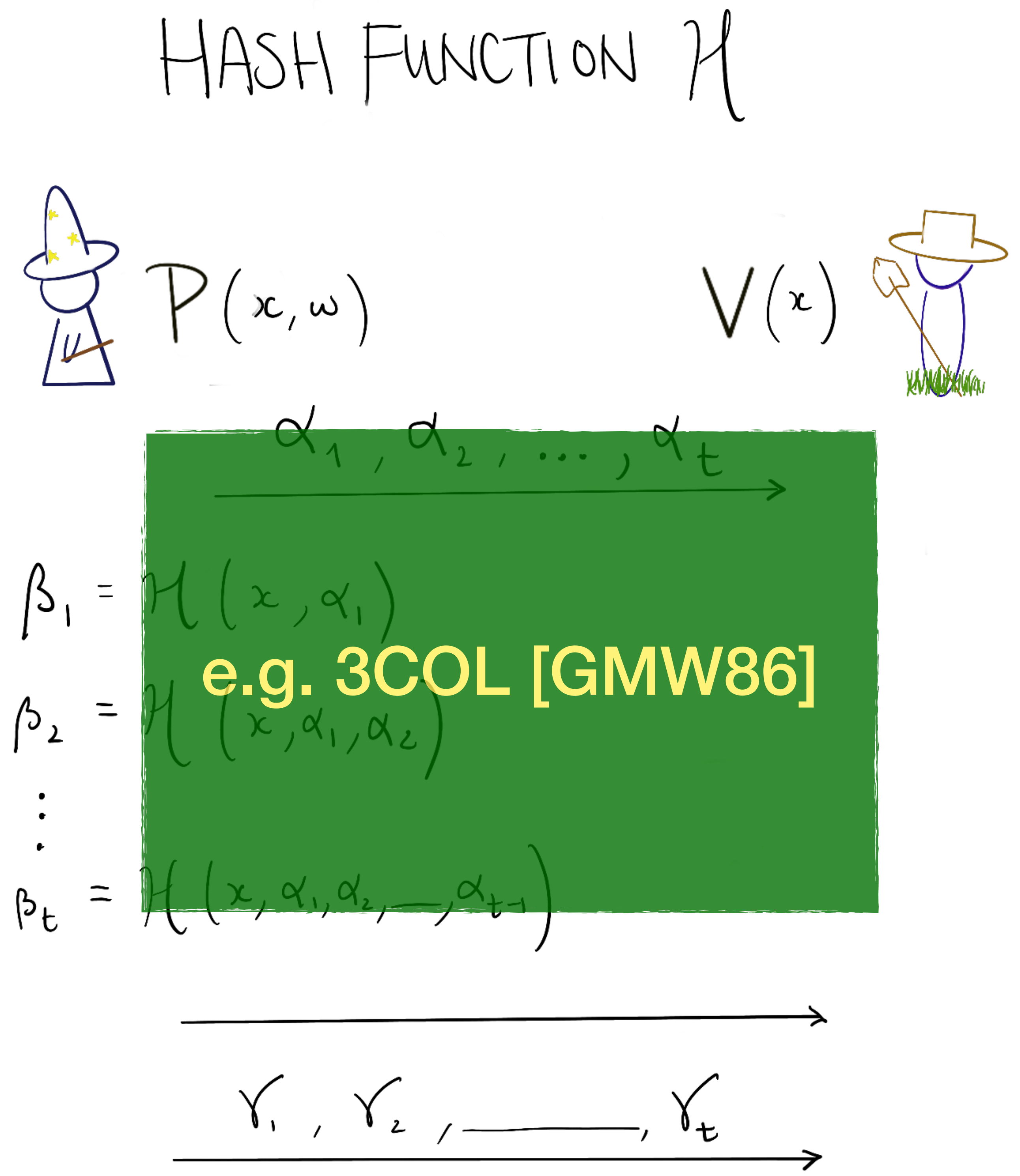

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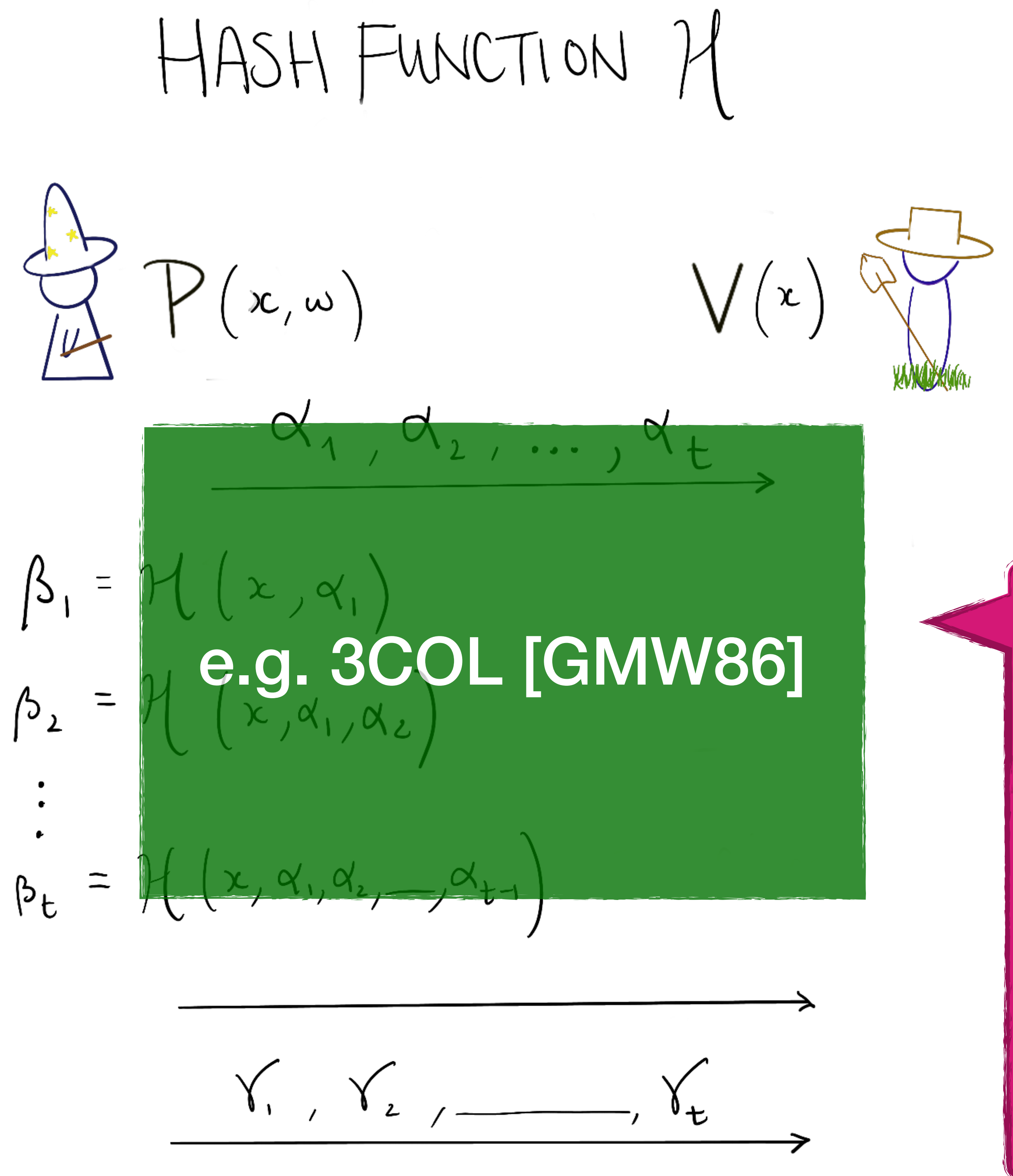

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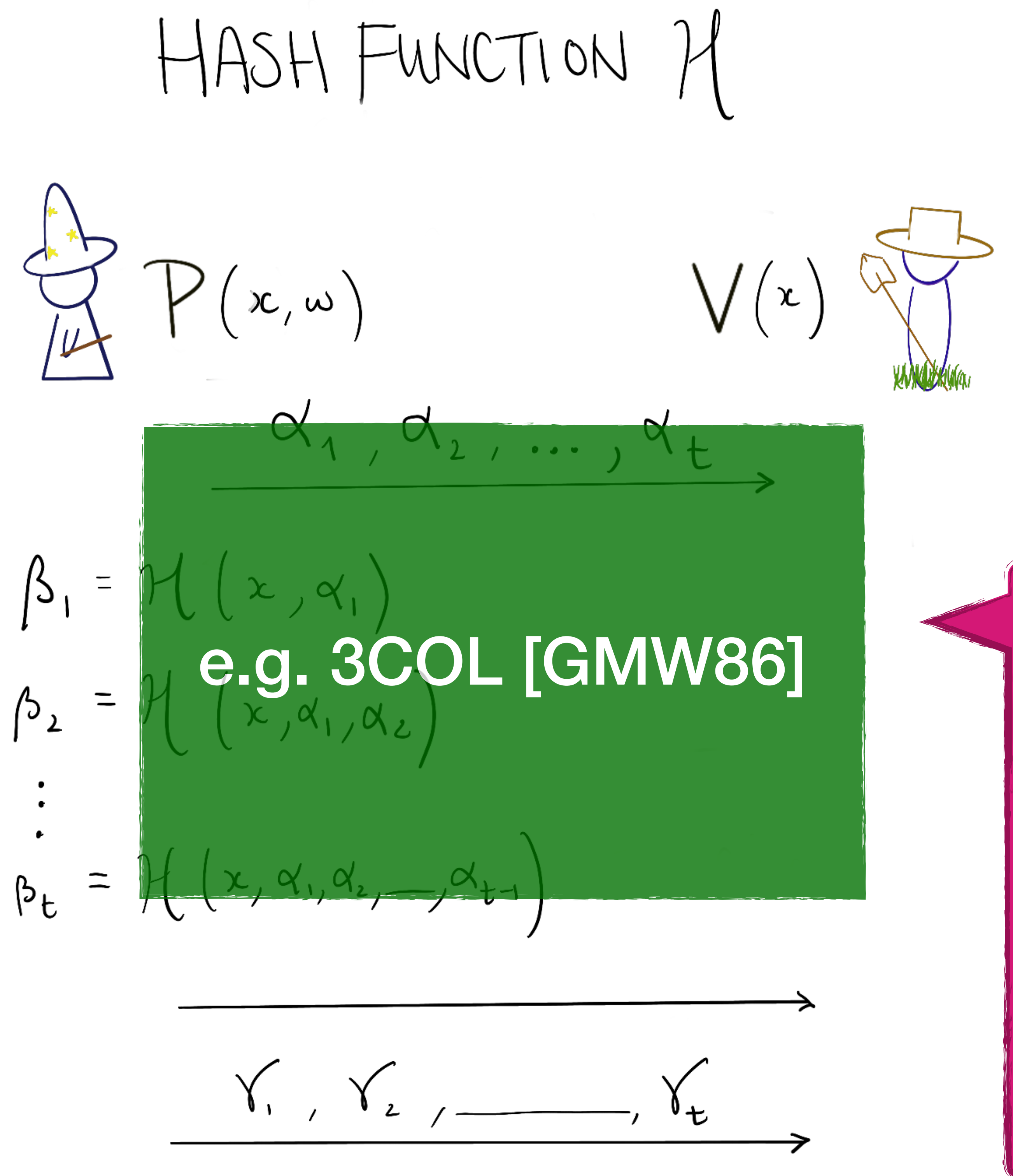
Expensive!



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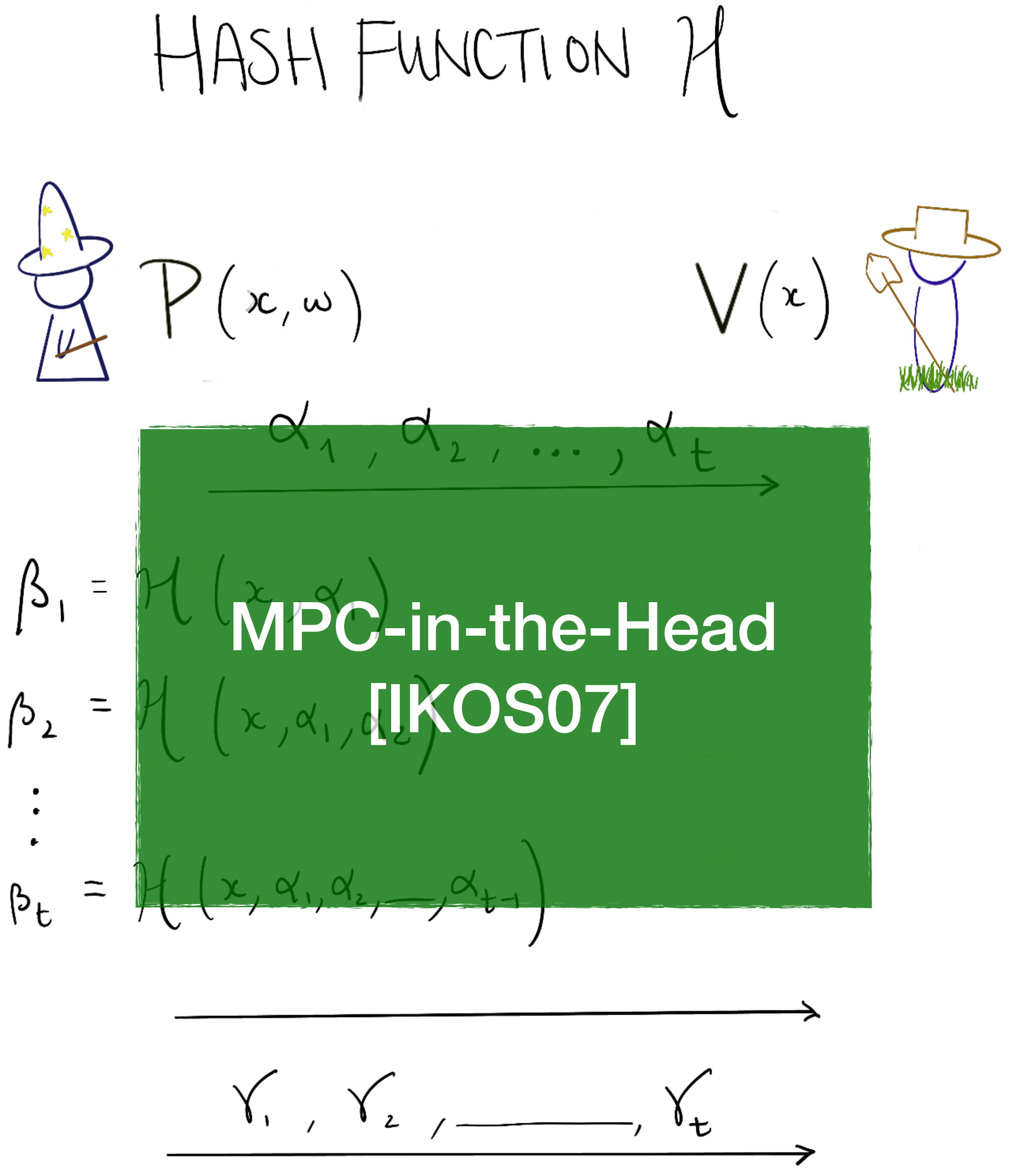
Our Work

We give an *efficient* (smaller proof size) base NIZK construction for NP from LWE *without* parallel repetition and Karp reductions.

NIZK Argument in the CRS model



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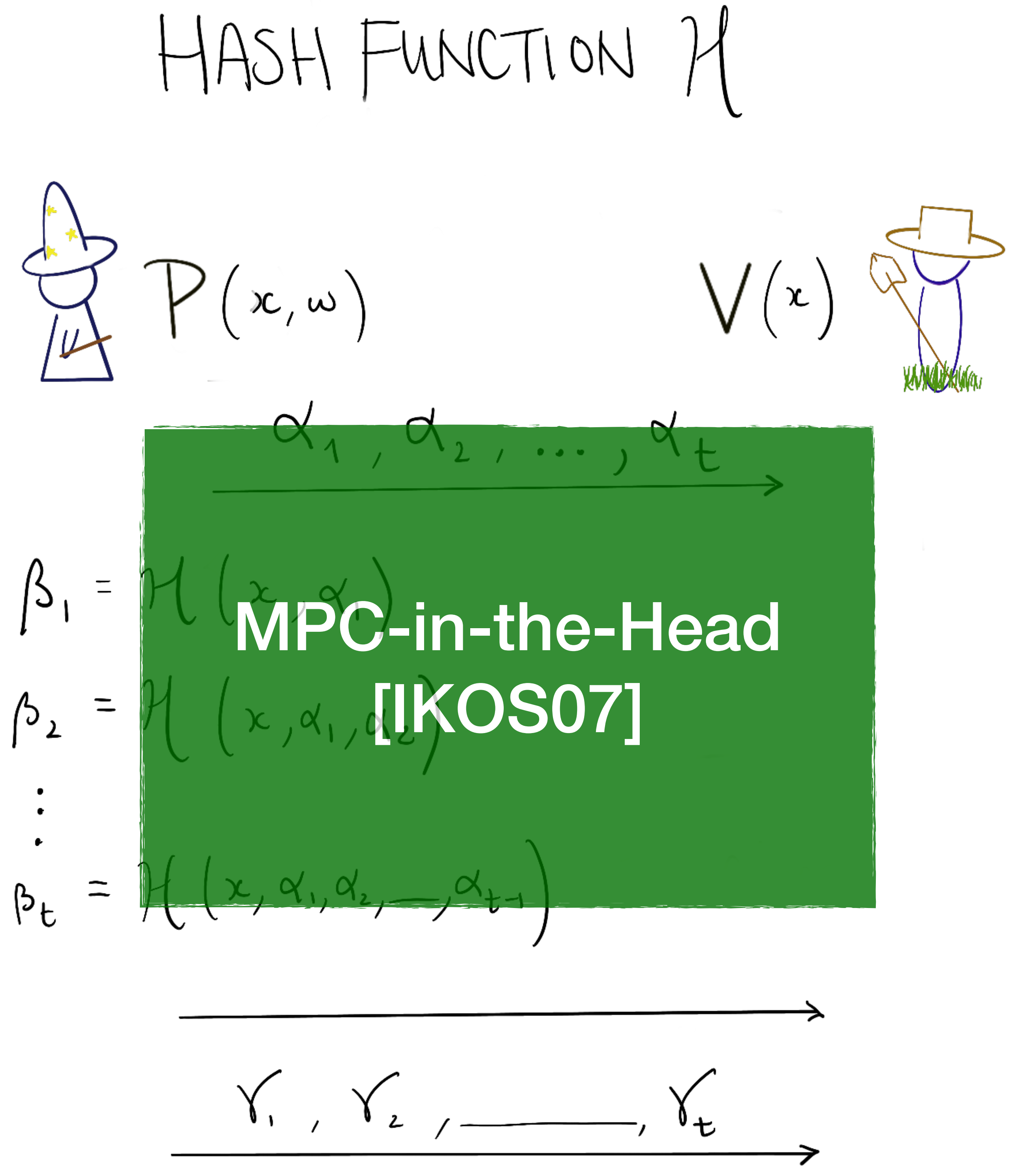

MPC-in-the-Head [IKOS07]

Allows us to translate work on efficient perfectly robust MPC protocols [DIK10, BGJK21, GPS21] to efficient NIZKs from LWE!

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[GGI+15] Can use FHE to bootstrap an underlying NIZK to one with proof size $|w| + \text{poly}(k)$ bits.

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We show that this yields less efficient proofs.

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- **Our work:** The bad challenge set structure present in a modification of the [IKOS07] protocol only needs *recurrent* list-recovery. Therefore, we can use *qualitatively simpler* codes (Reed-Solomon codes concatenated with *multiple* random codes) and directly use polynomial reconstruction [Sud97, GS98] to achieve an improved block size of $\tilde{O}(k)$.

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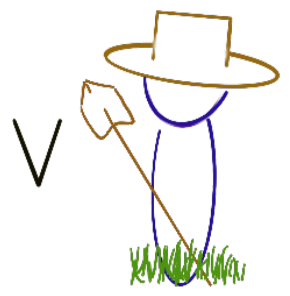

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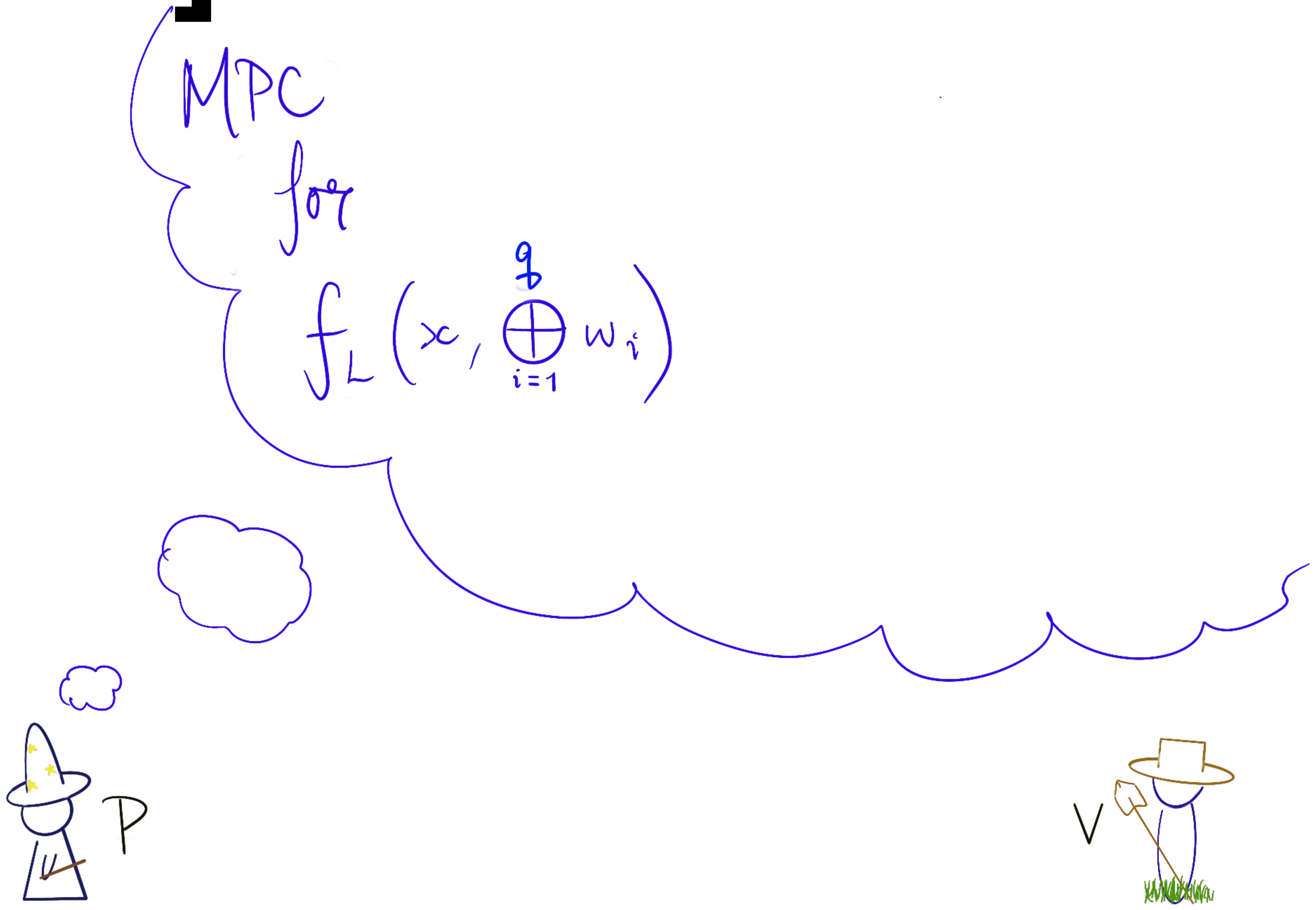

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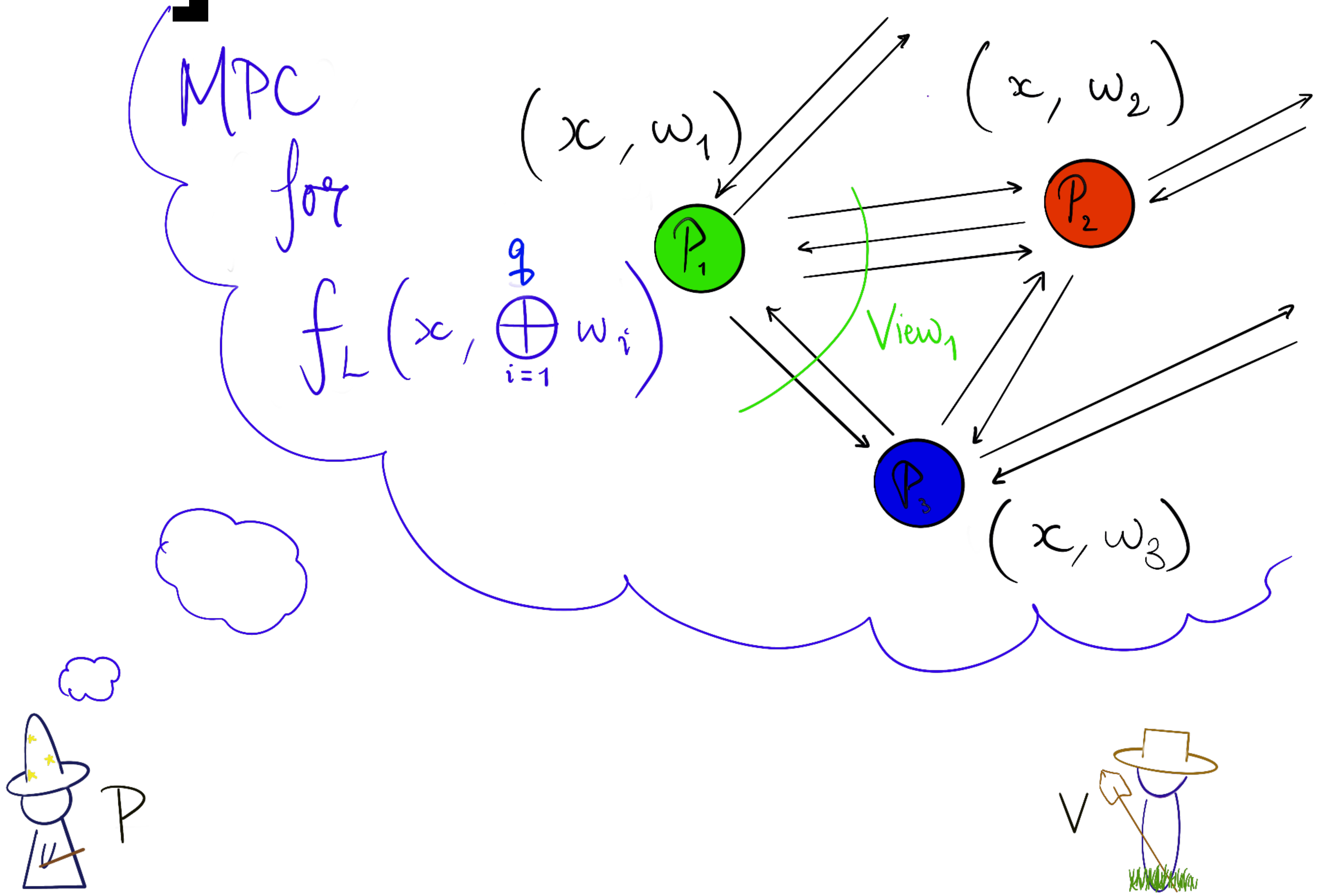

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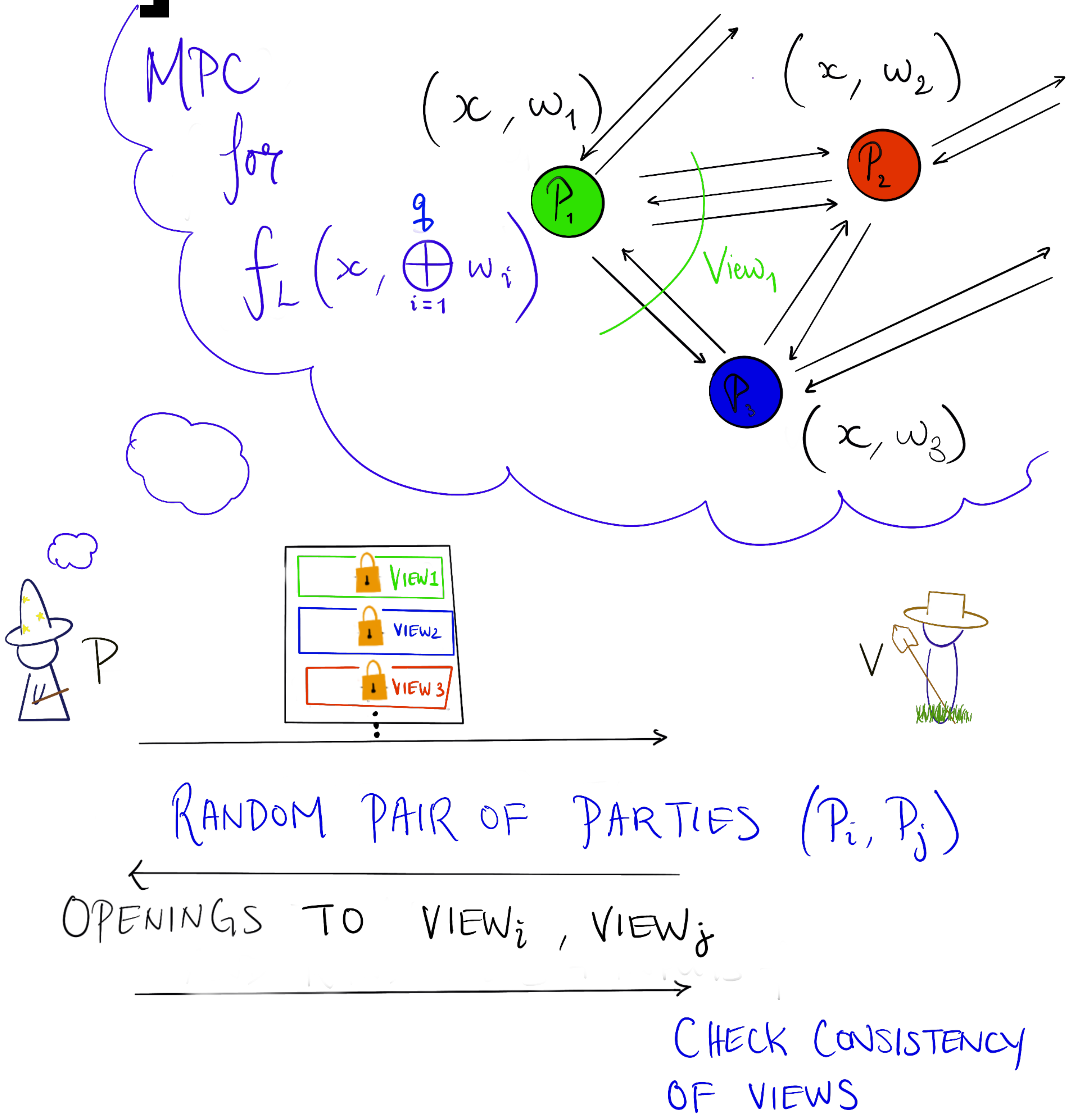


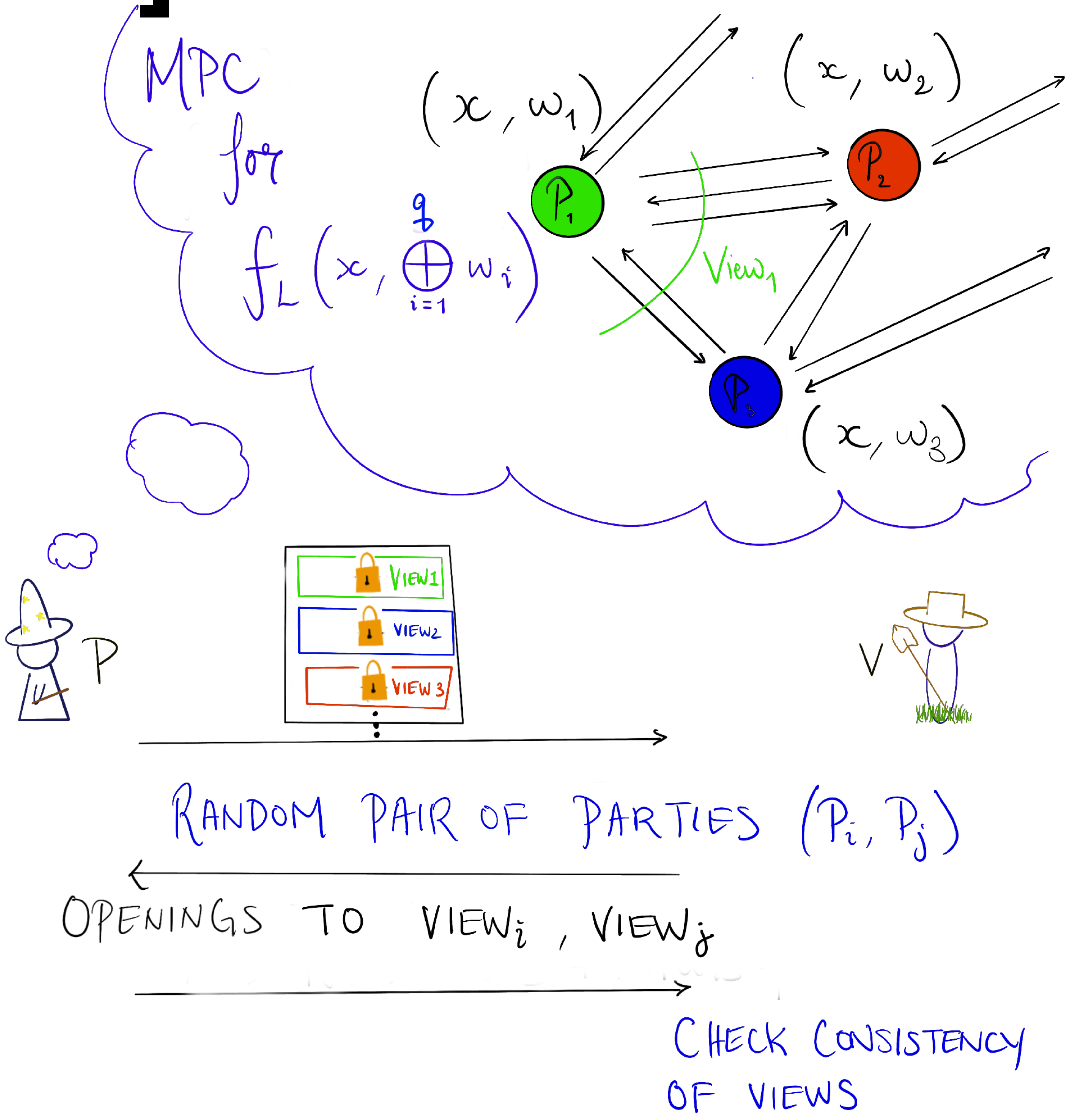
Black-box use of the MPC protocol!

MPC-in-the-Head [IKOS07]

View of $P_1(x, w_1 ; r)$

1. $m_1 \longrightarrow P_2$

2. $m_2 \longleftarrow P_3$



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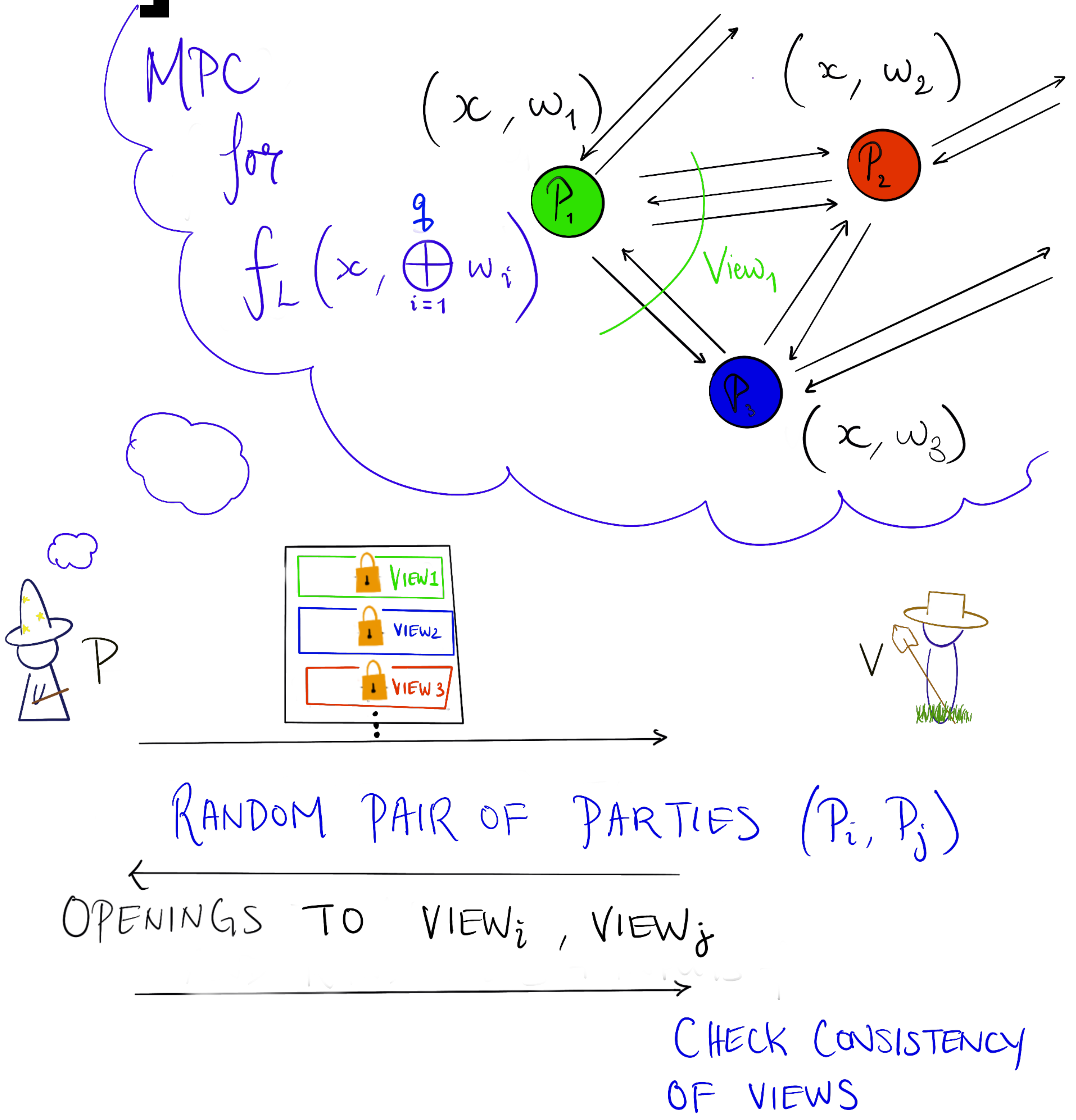
$\Downarrow$ $\text{NEXT}(1, x, w_1, r, m_2)$



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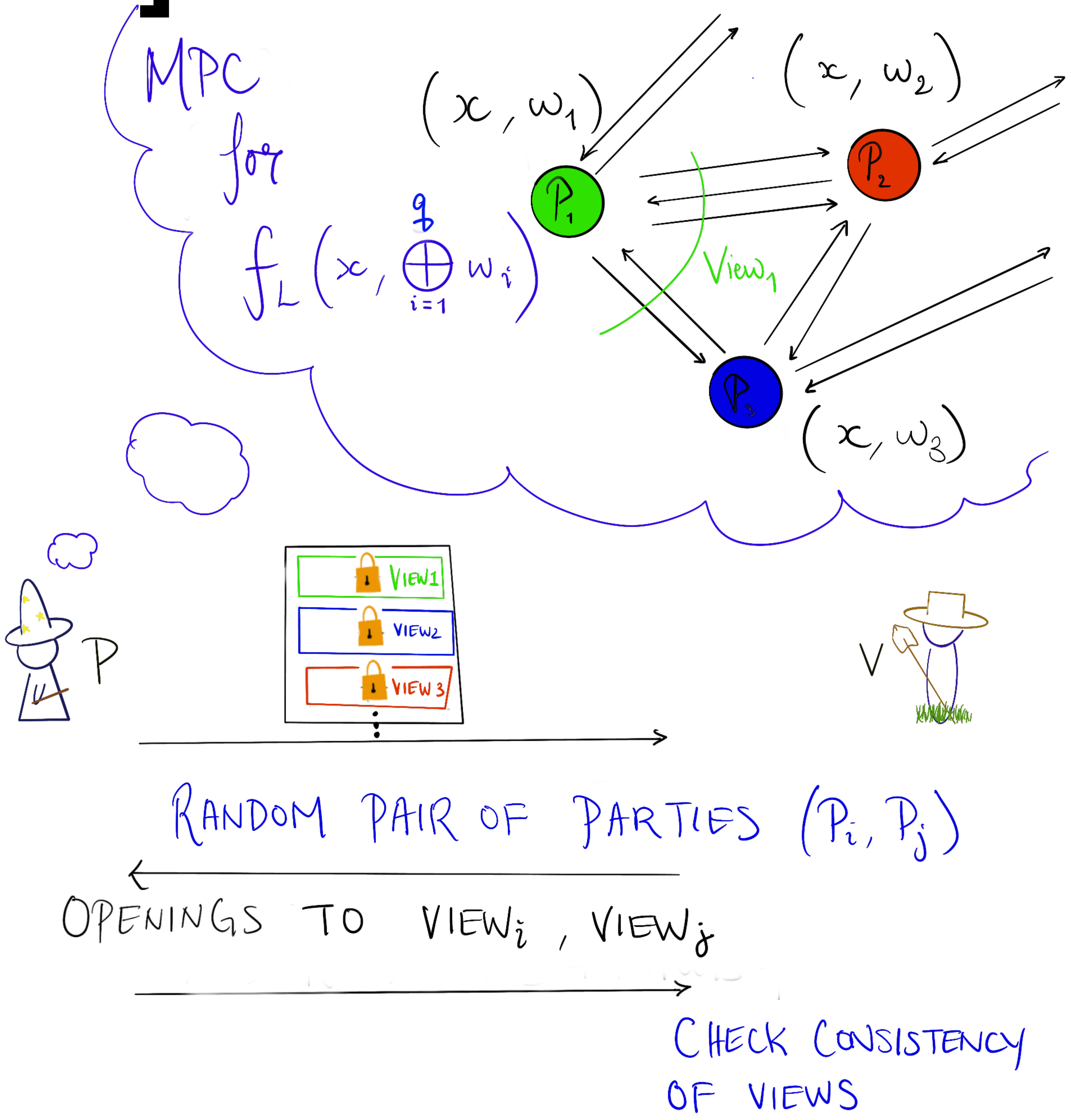

Our Modification of MPC-in-the-Head

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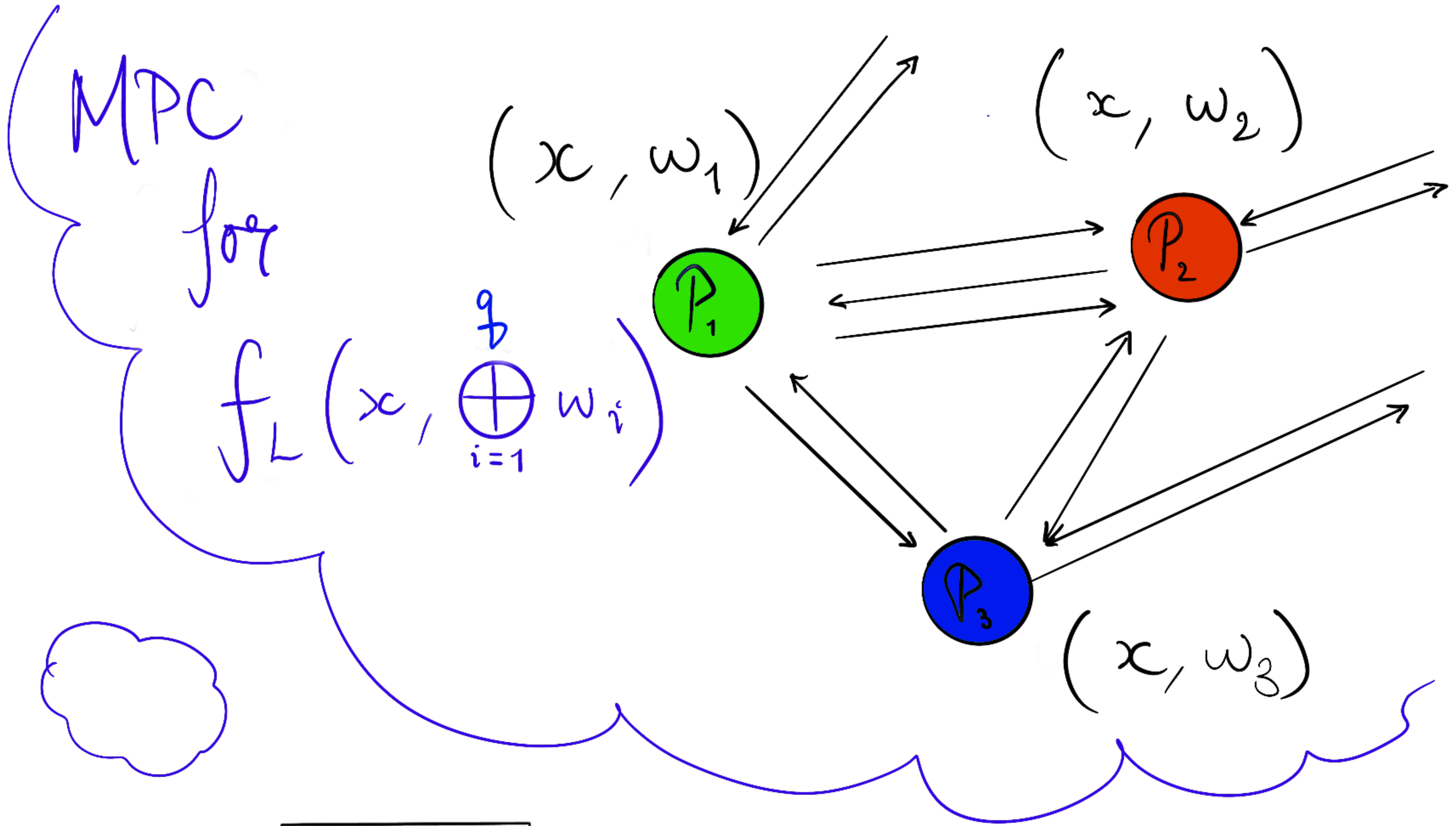
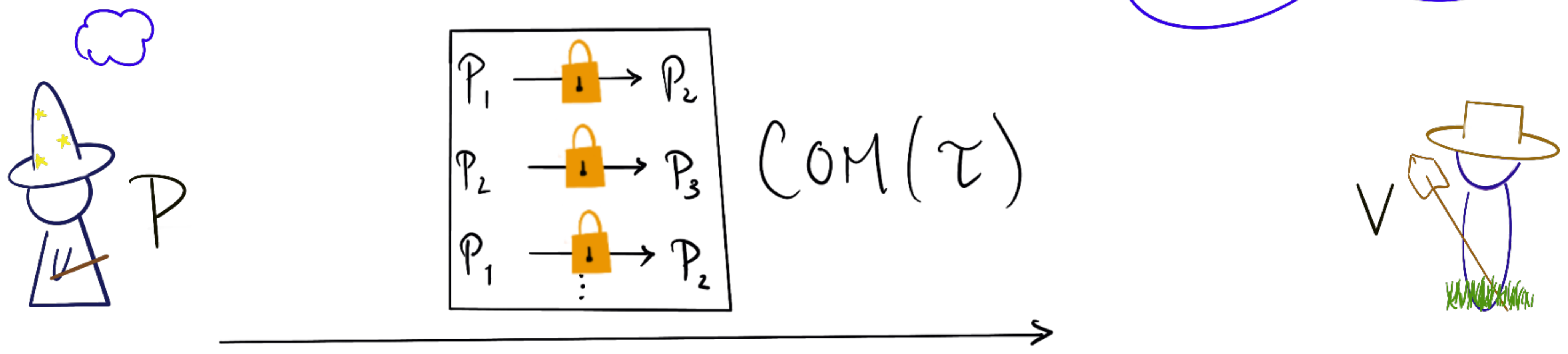
Non-black-box use of the MPC protocol!





RANDOM PARTY P_i

OPENINGS TO ALL INCIDENT MSGS AND RANDOMNESS + INPUTS for P_i

USE NEXT$(\cdot)$ TO CHECK CONSISTENCY

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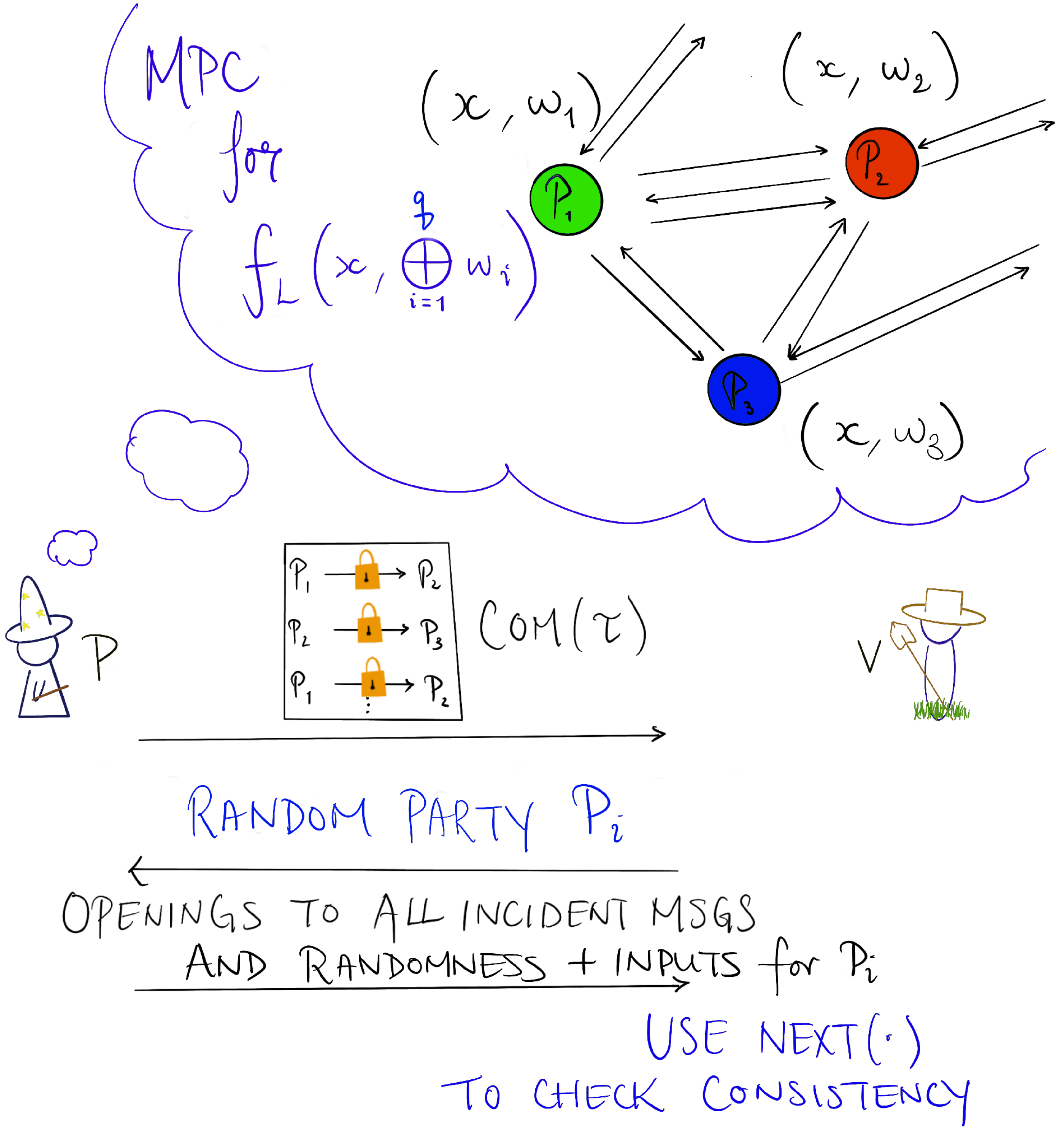

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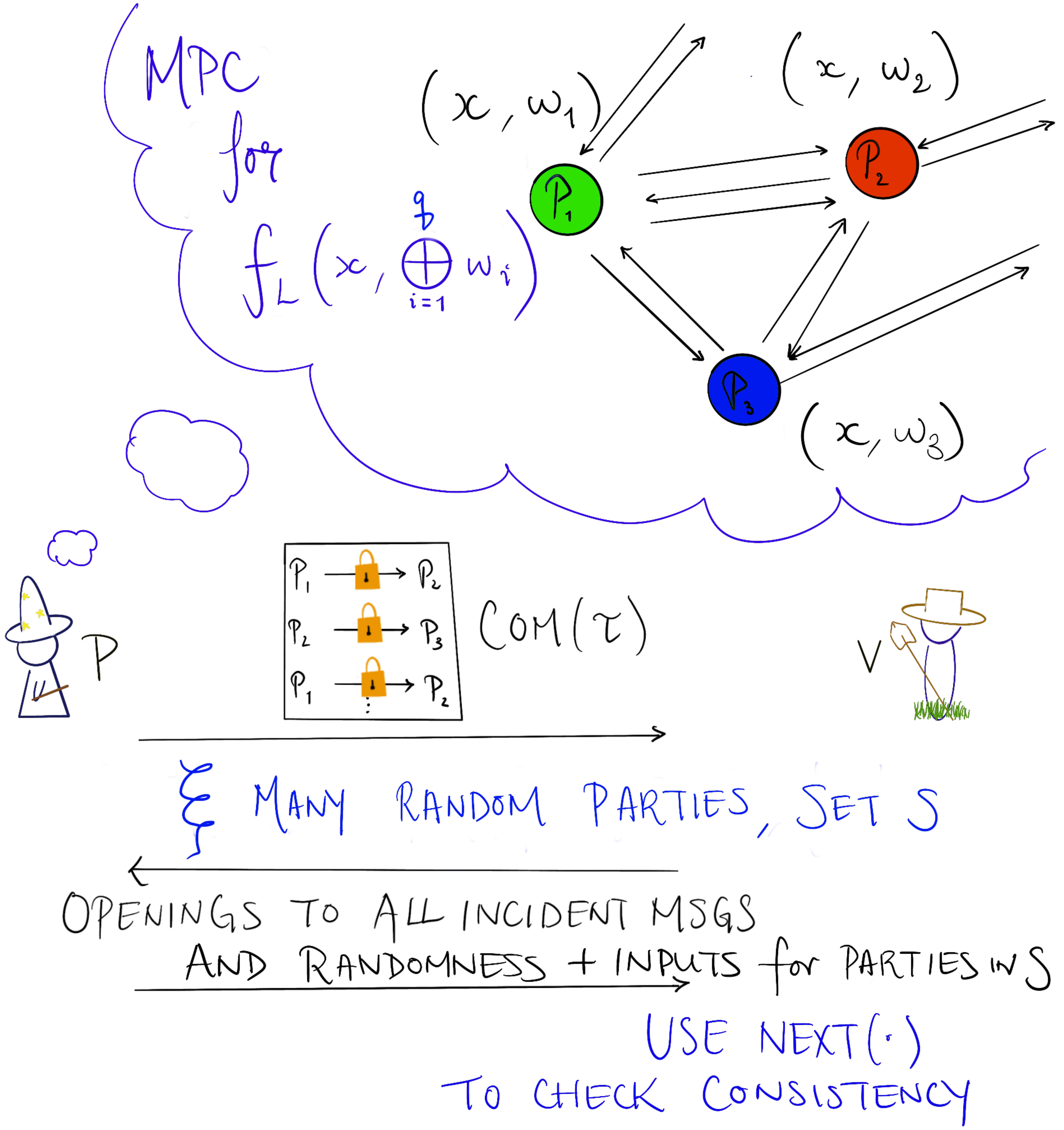

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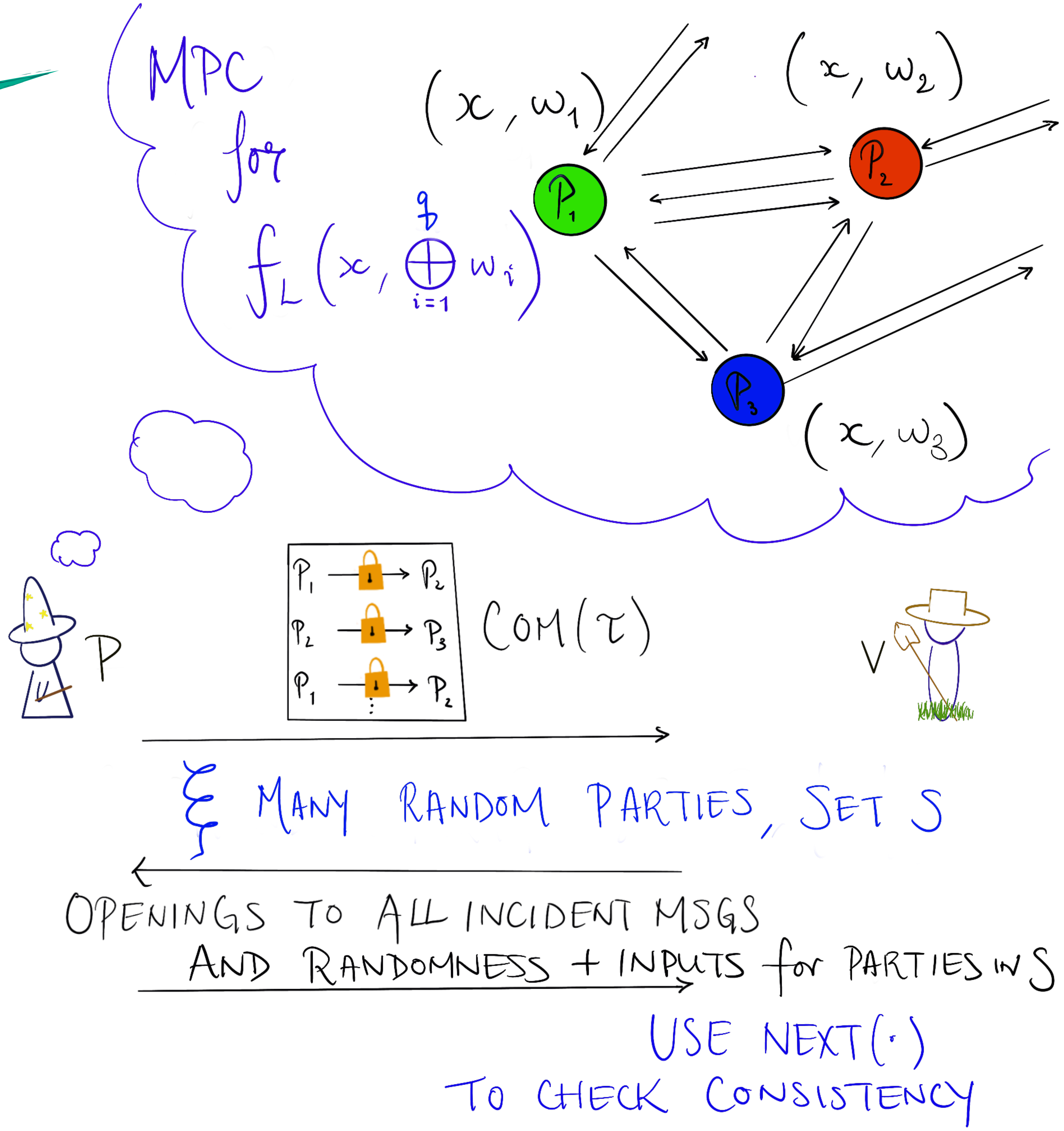

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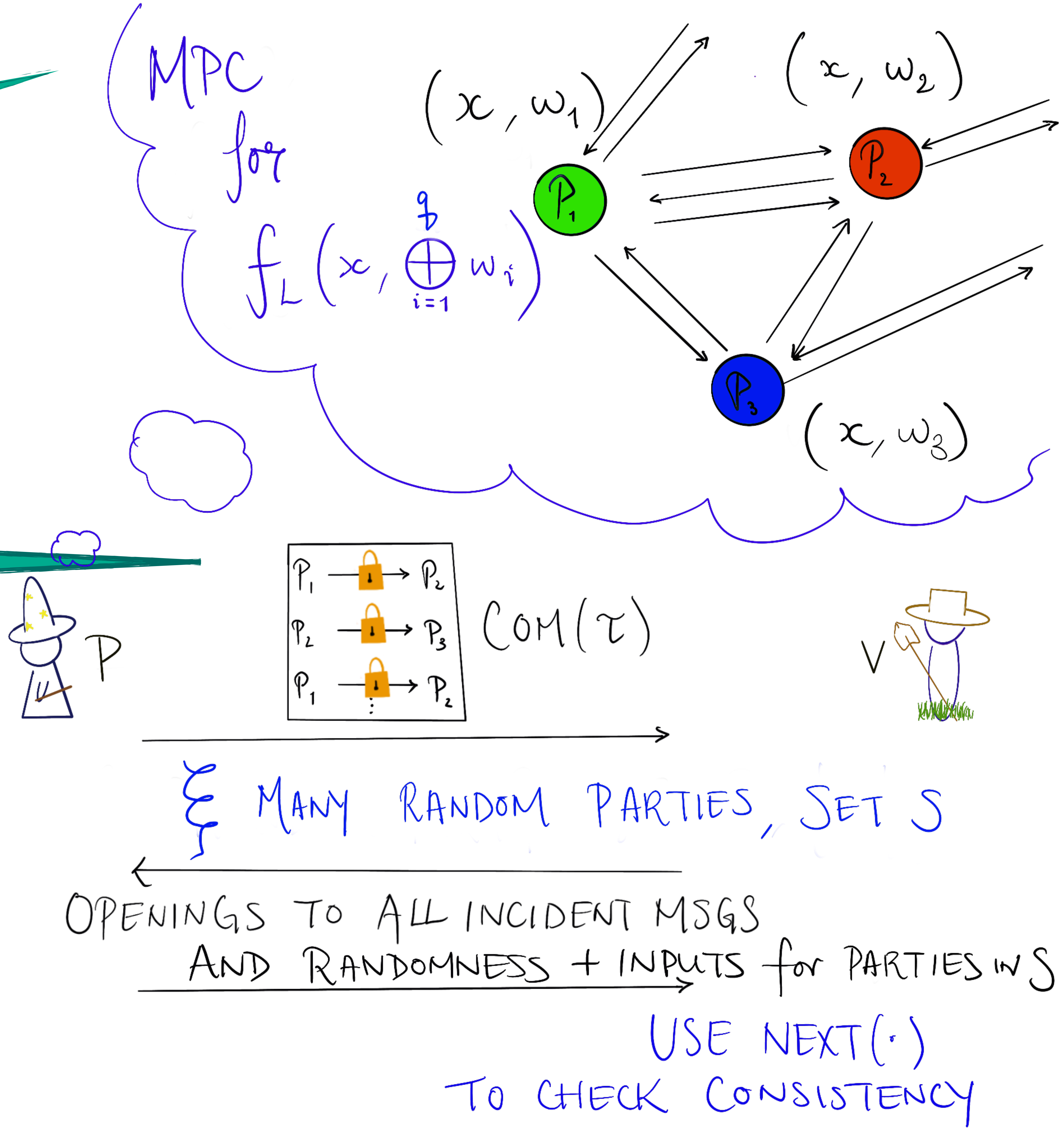

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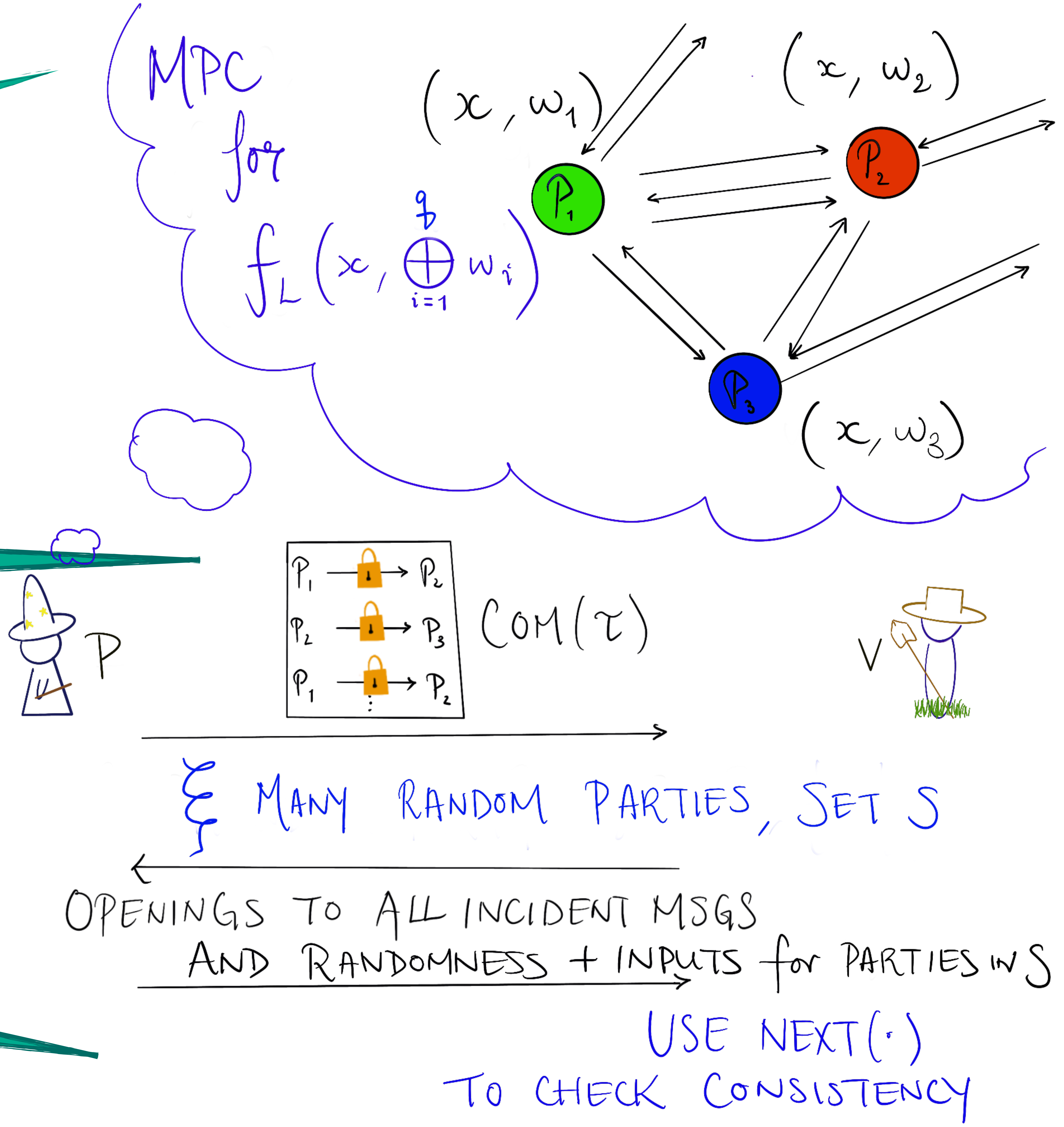
Each party's view is now *independently* verifiable!



A Coding-Theoretic Instantiation of Fiat-Shamir following [HLR21]

Amplifying Soundness via Parallel Repetition

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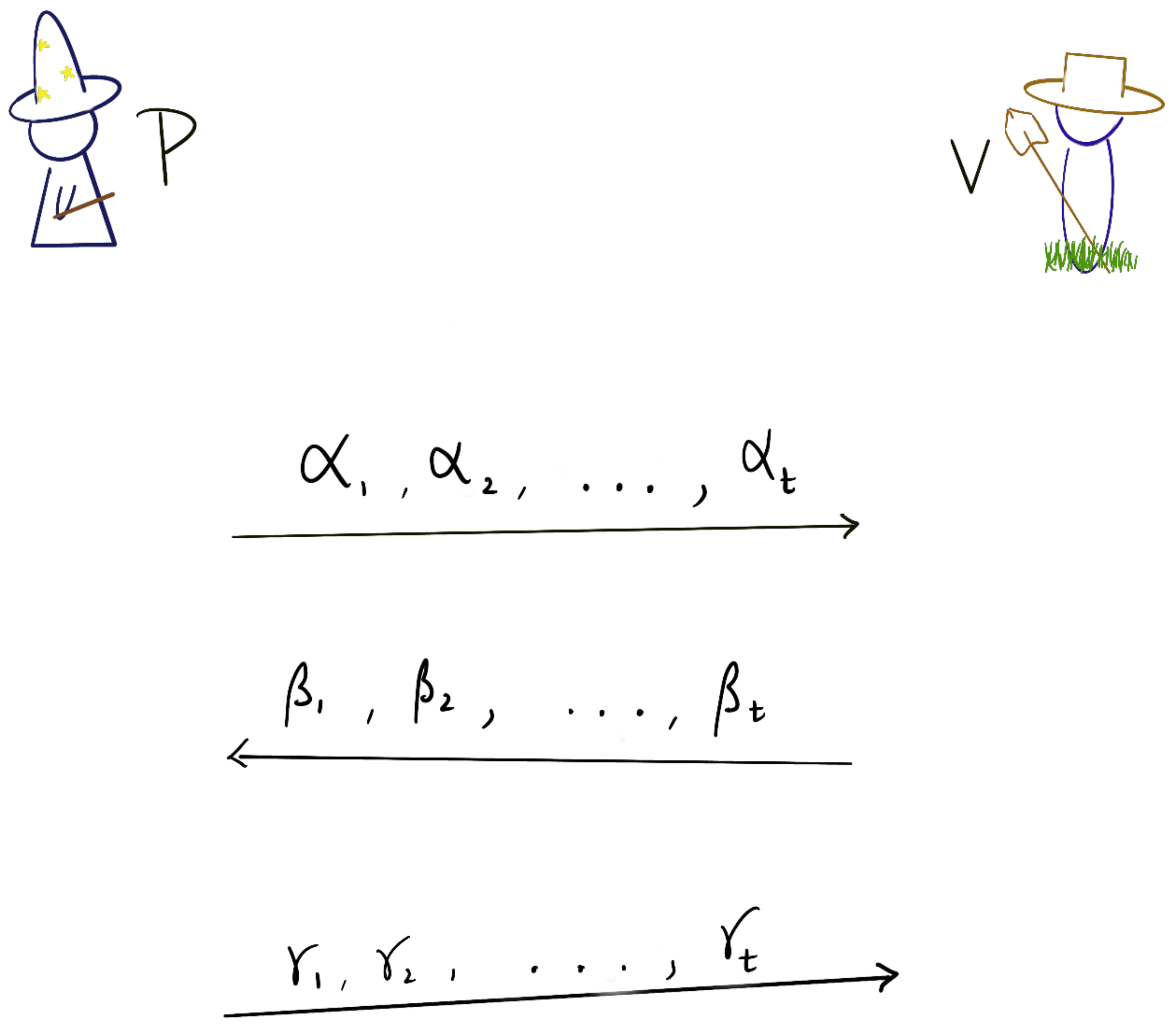

Consider an interactive proof for some NP language L that satisfies:

- Completeness
- $negl$-soundness against unbounded provers (statistical soundness)
- Honest-verifier zero-knowledge (HVZK)
- Public coin

Fiat-Shamir Paradigm [FS87]

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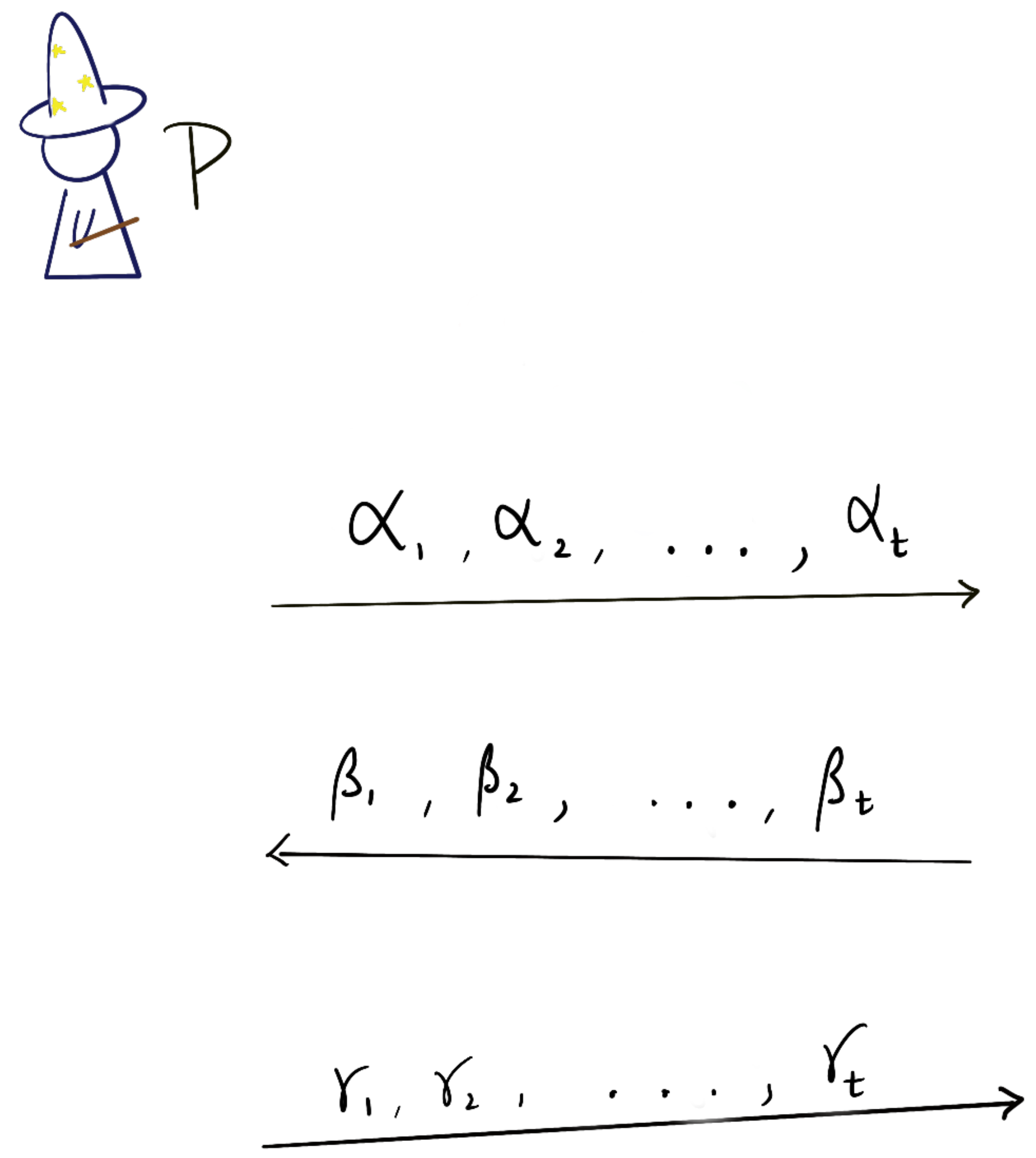

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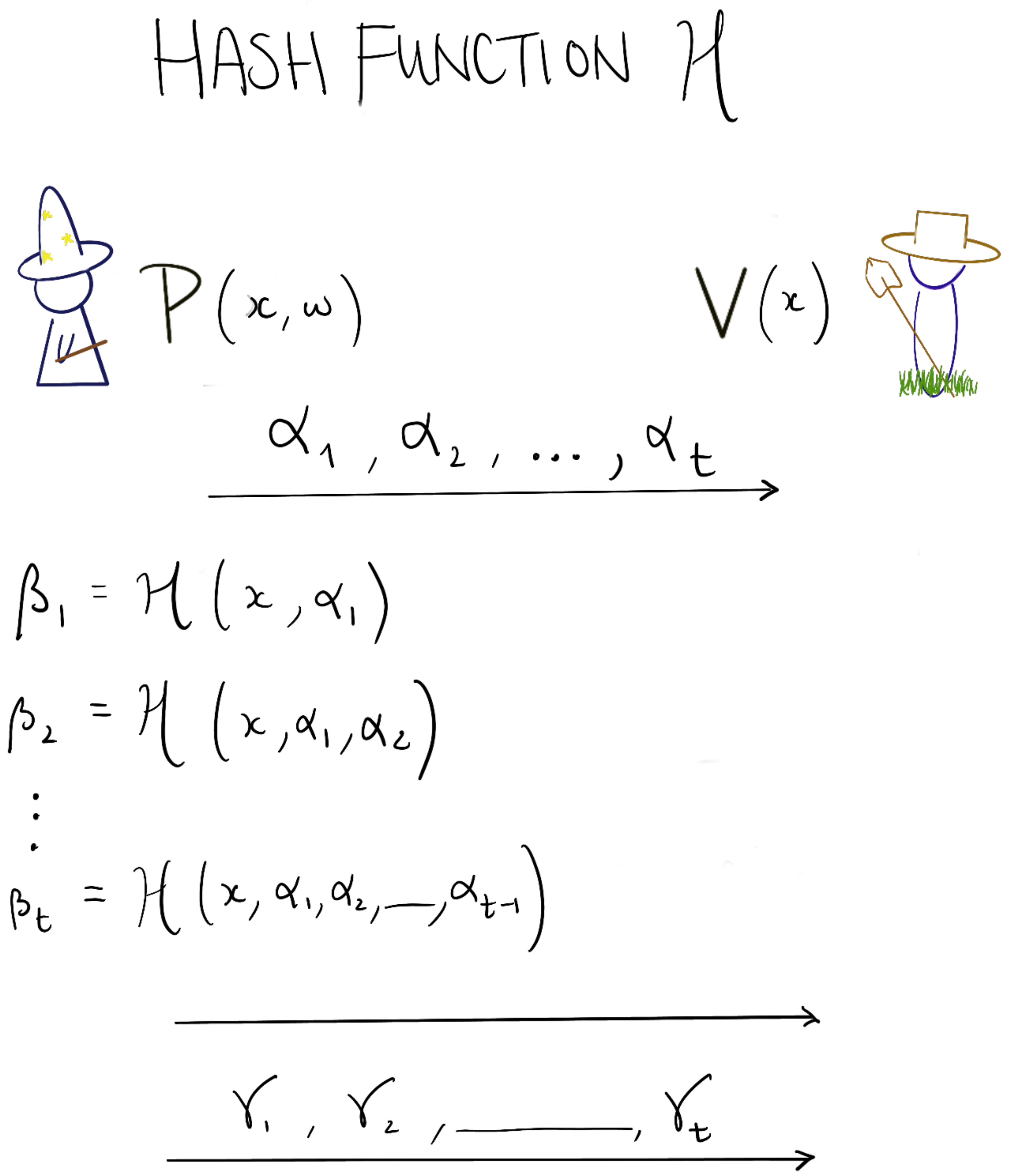

Correlation Intractability [CGH04]

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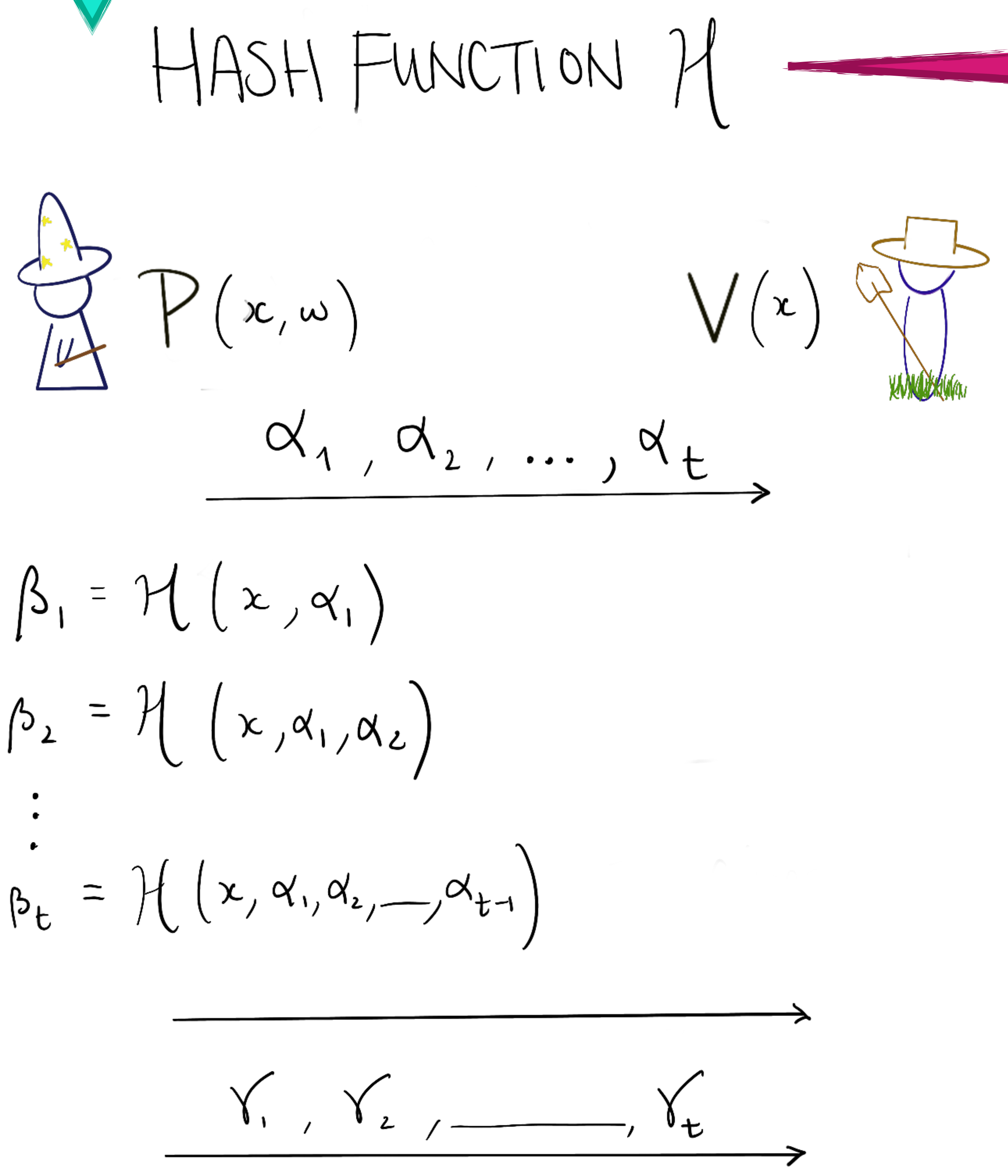

Soundness is preserved if H is sampled from a correlation intractable hash family for an appropriate relation R.

[CGH04] **Def'n:** A hash family $\mathscr{H}$ is *correlation intractable* (CI) for a sparse relation R if for all PPT $\mathscr{A}$

$$Pr_{\substack{h \leftarrow \mathscr{H} \\ x \leftarrow \mathscr{A}(h)}} \left[(x, h(x)) \in R\right] = \text{negl}$$

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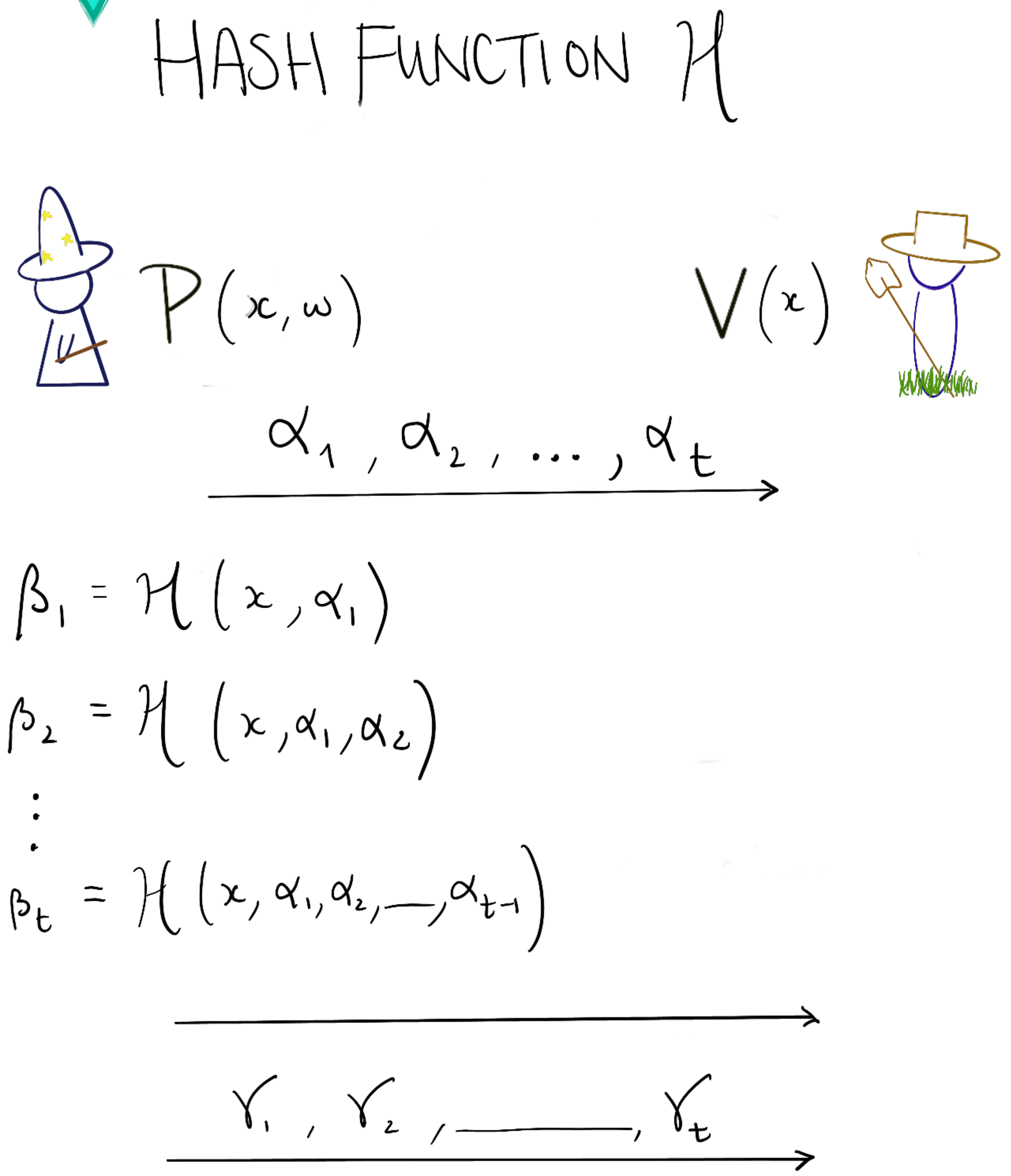

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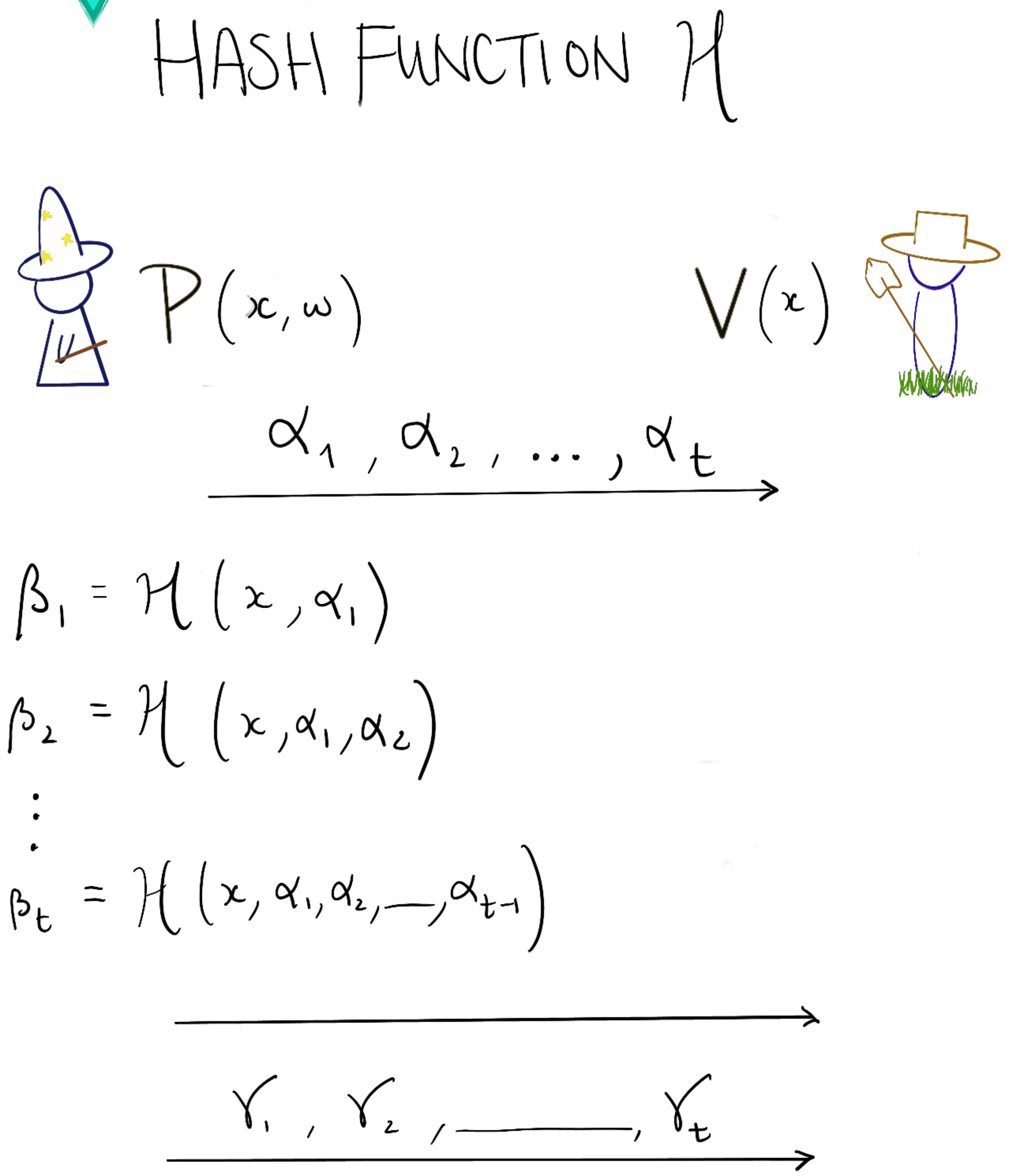

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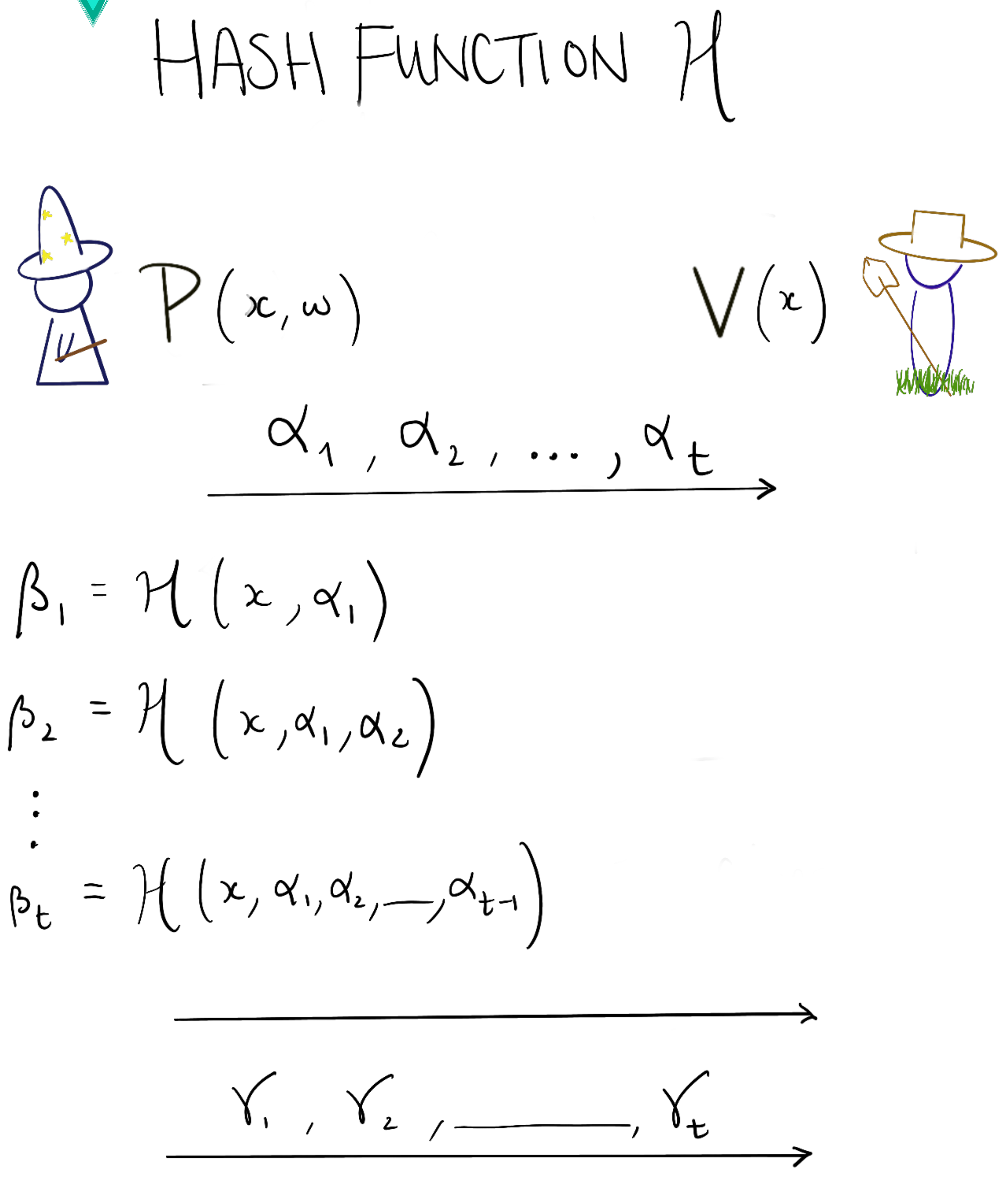

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[CCH+19] “*Bad Challenges*” (there’s some response that fools V into accepting)

Fiat-Shamir from Coding Theory [HLR21]

Parallel repetition gives a bad challenge set with a nice combinatorial structure.

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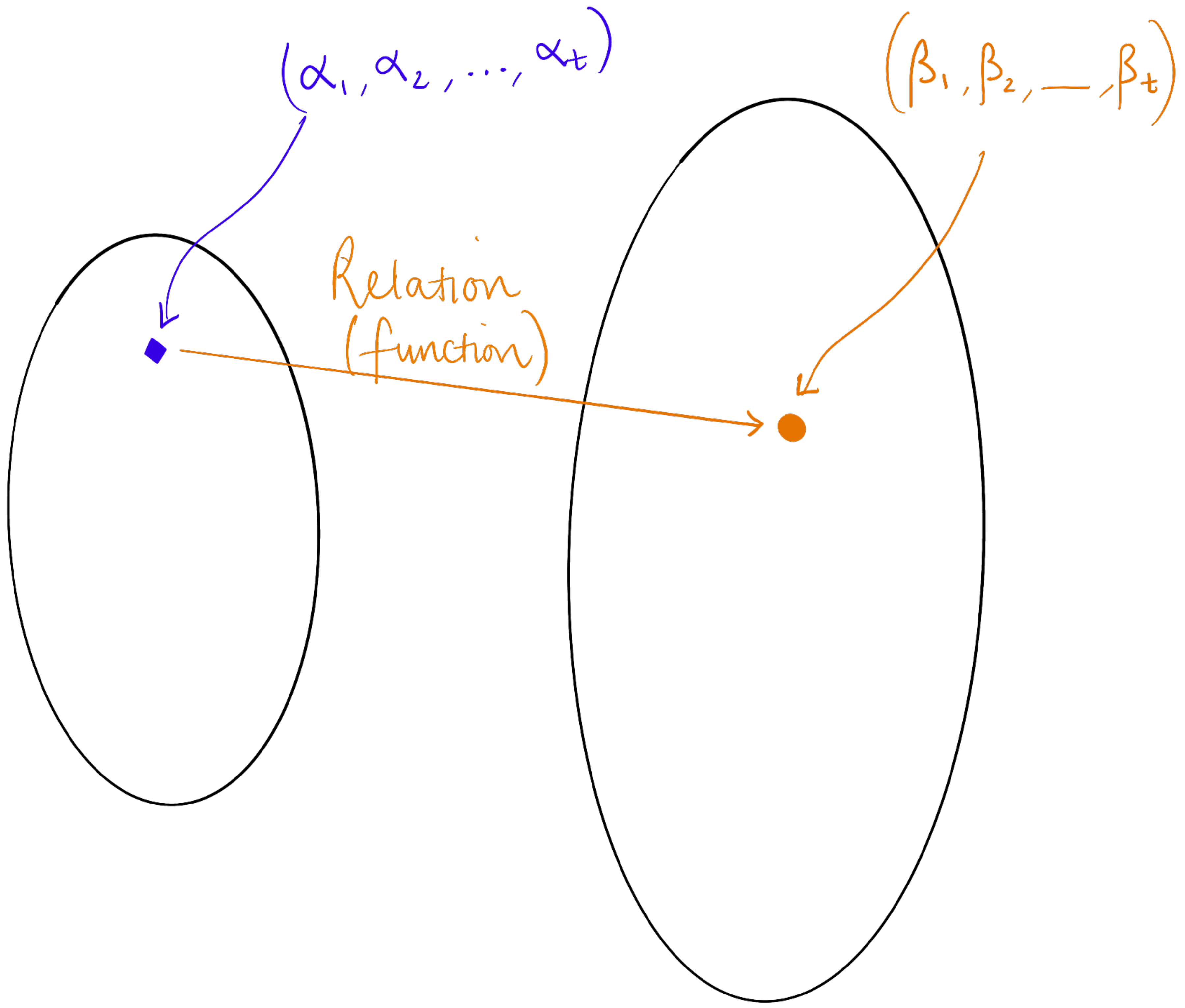
[PS19] addresses the case of functions.



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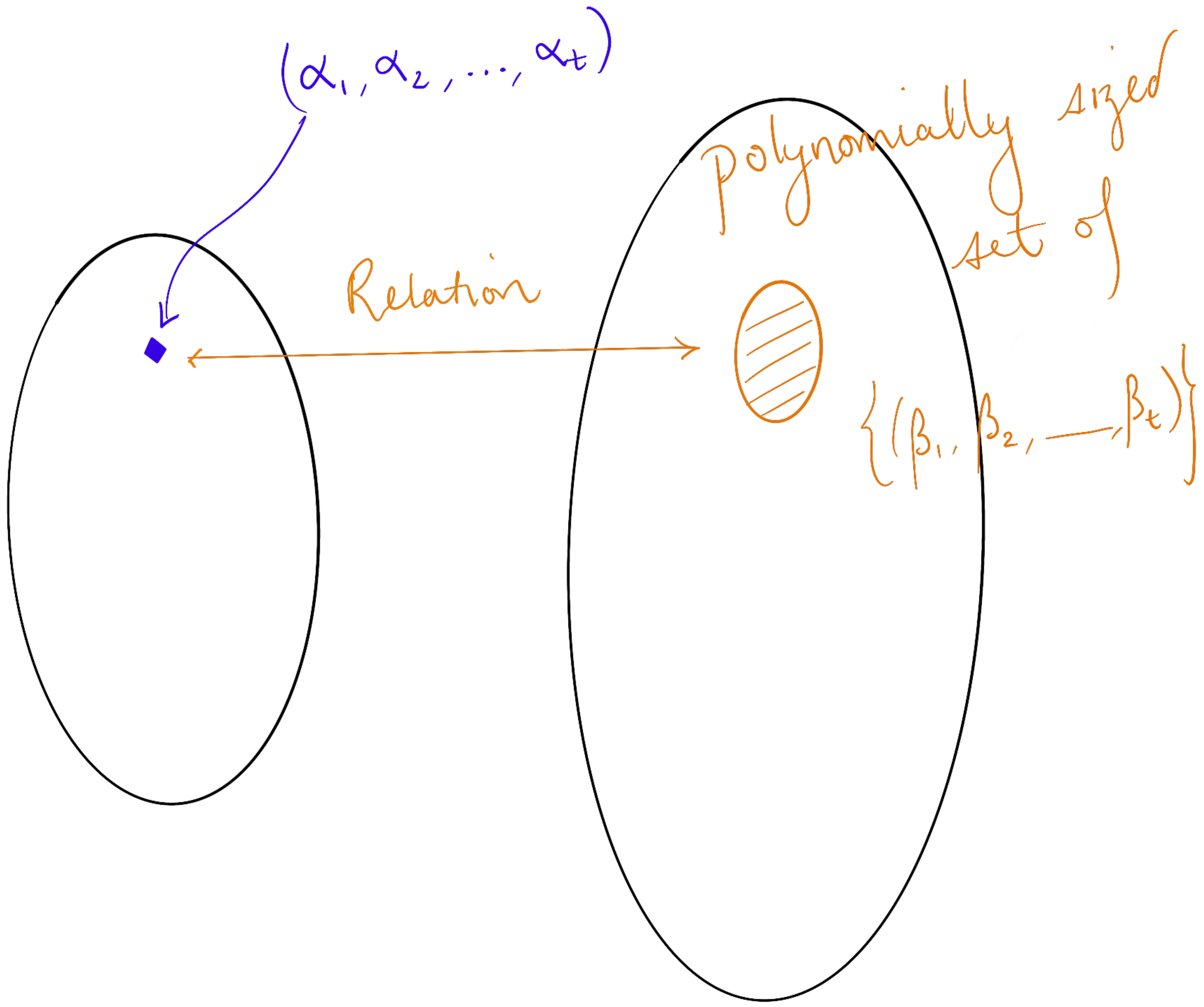
By a guessing reduction, [CCH+19, PS19] also addresses the case of polynomially many bad challenges.



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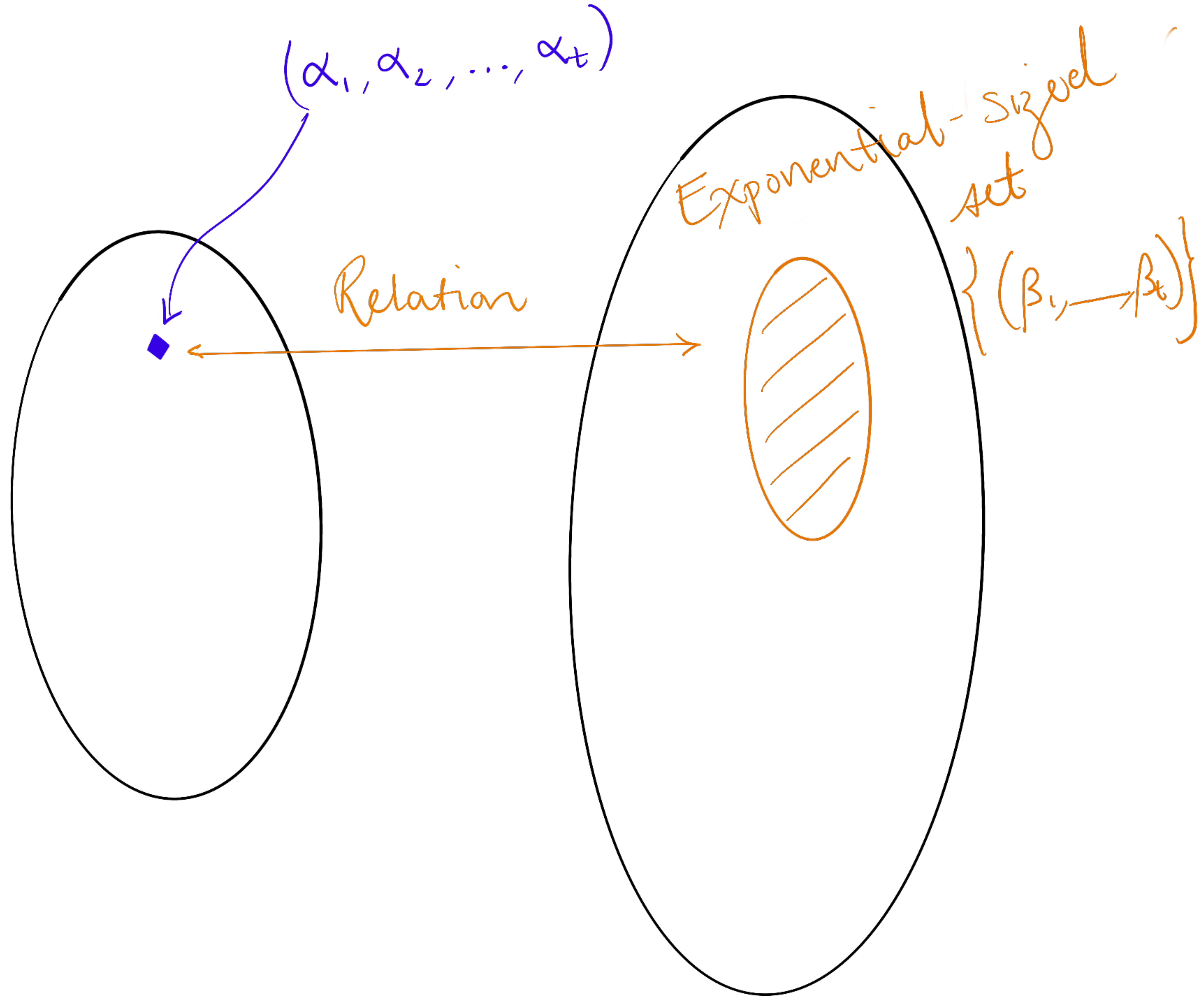
Too many bad challenges for the techniques of [CCH+19, PS19].



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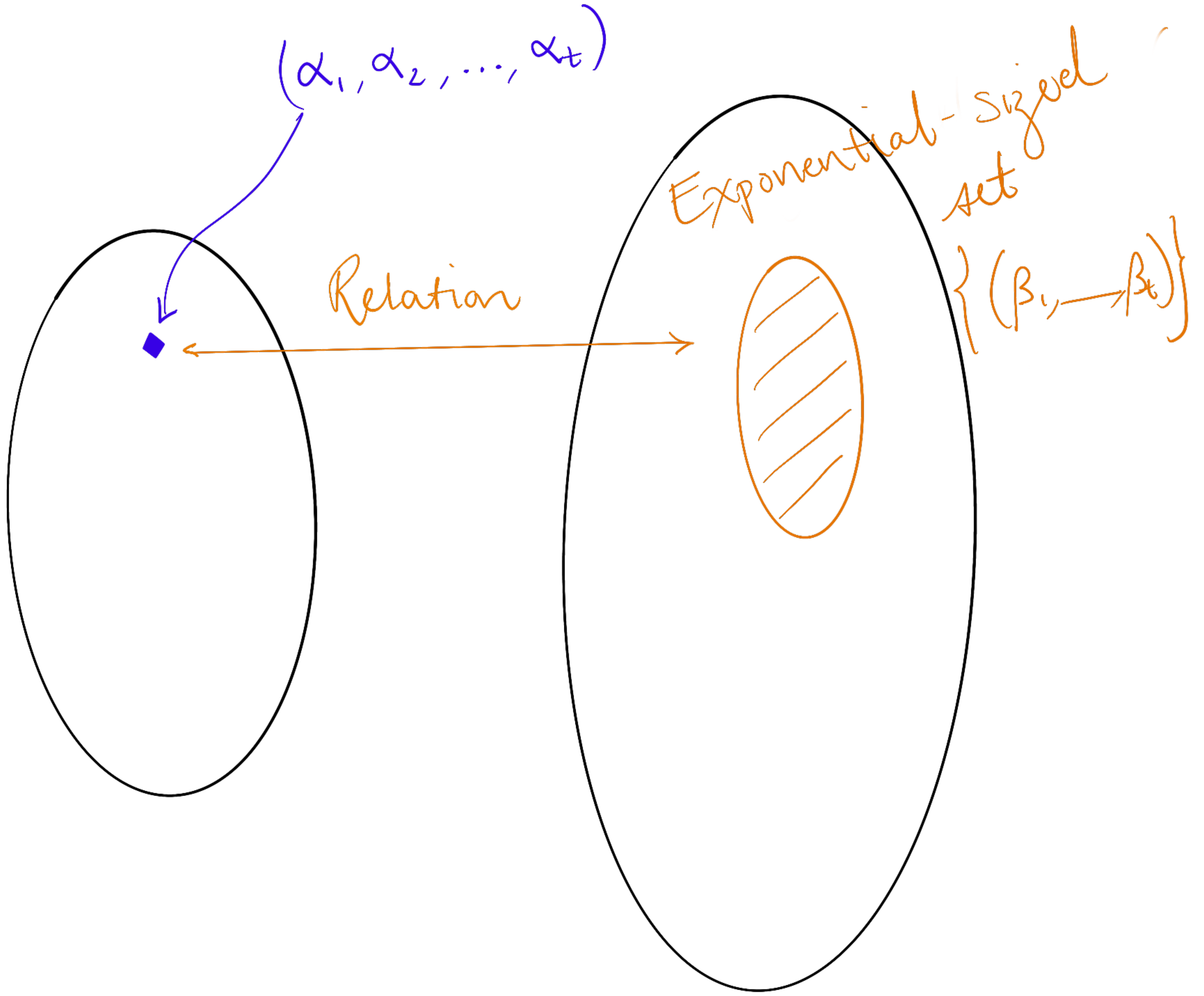
[HLR21] Use the product structure!



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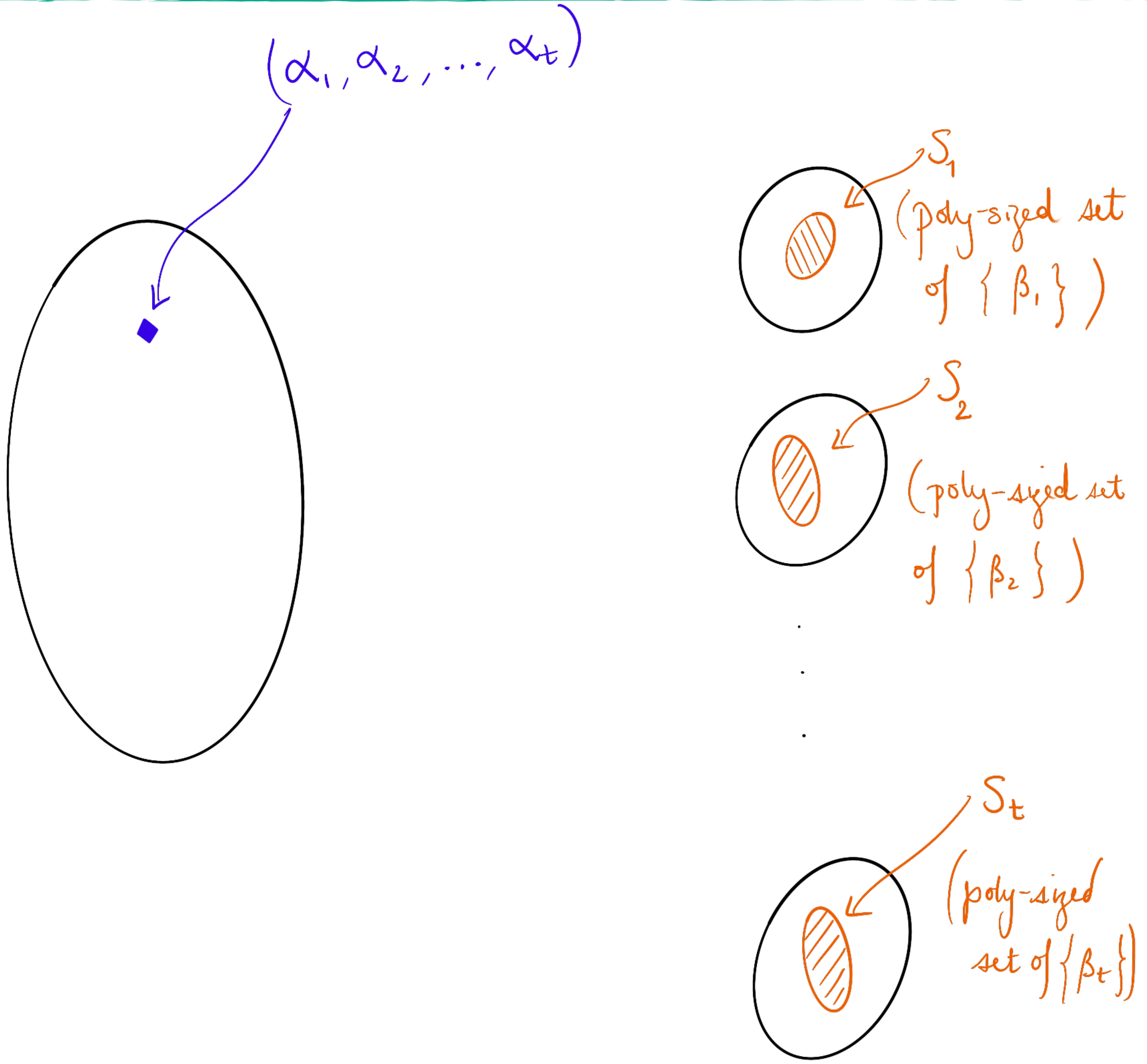
[HLR21] Use the product structure!



Fiat-Shamir from Coding Theory [HLR21]

Parallel repetition gives a bad challenge set with a nice combinatorial structure.

For a statement $x \notin L$:

$$R_x = \left\{ \left((\alpha_1, \ldots, \alpha_t), (\beta_1, \ldots, \beta_t) \right) : \exists (\gamma_1 \ldots, \gamma_t) \text{ s.t. } V(x, \vec{\alpha}, \vec{\beta}, \vec{\gamma}) = 1 \right\}$$

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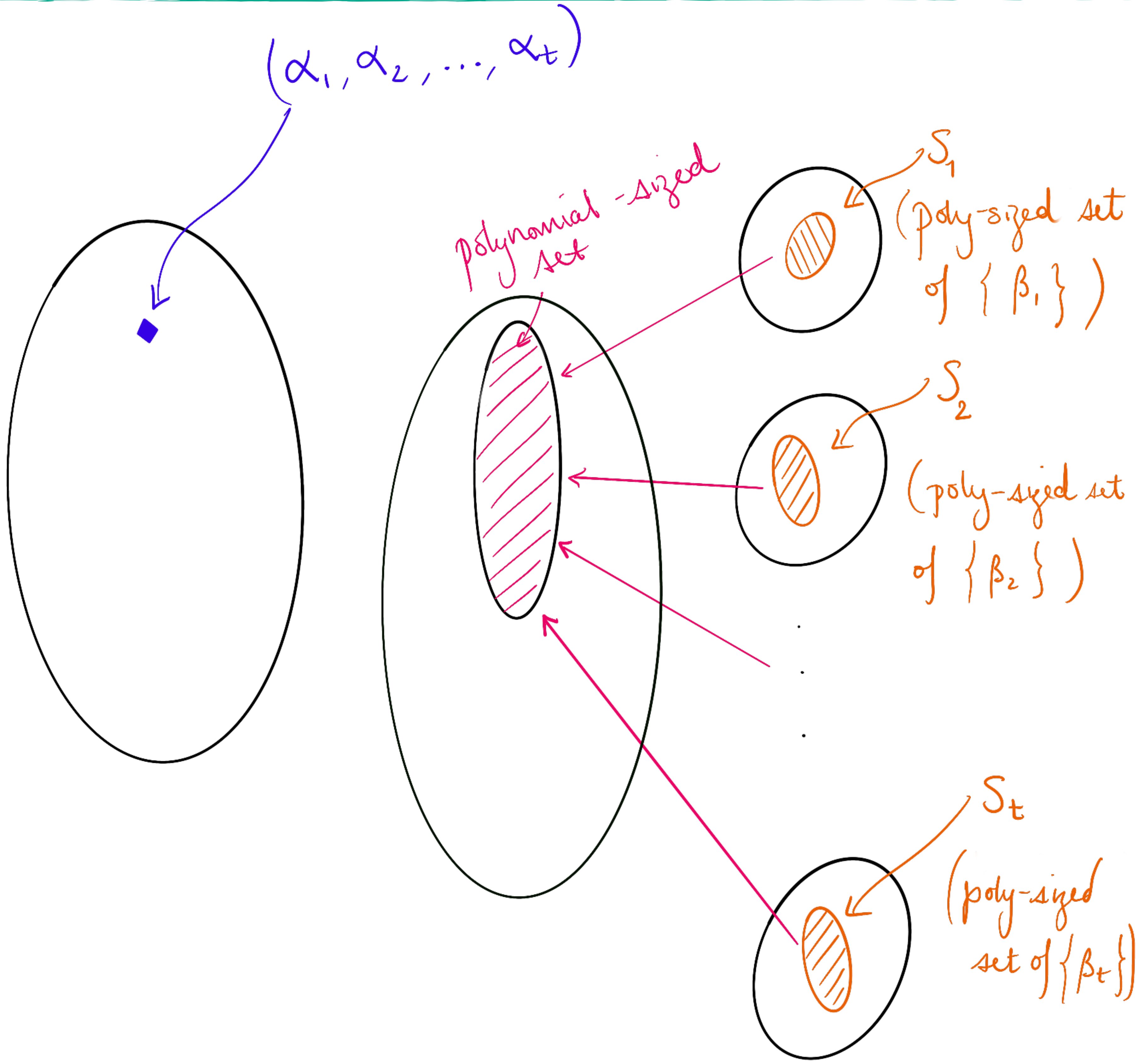

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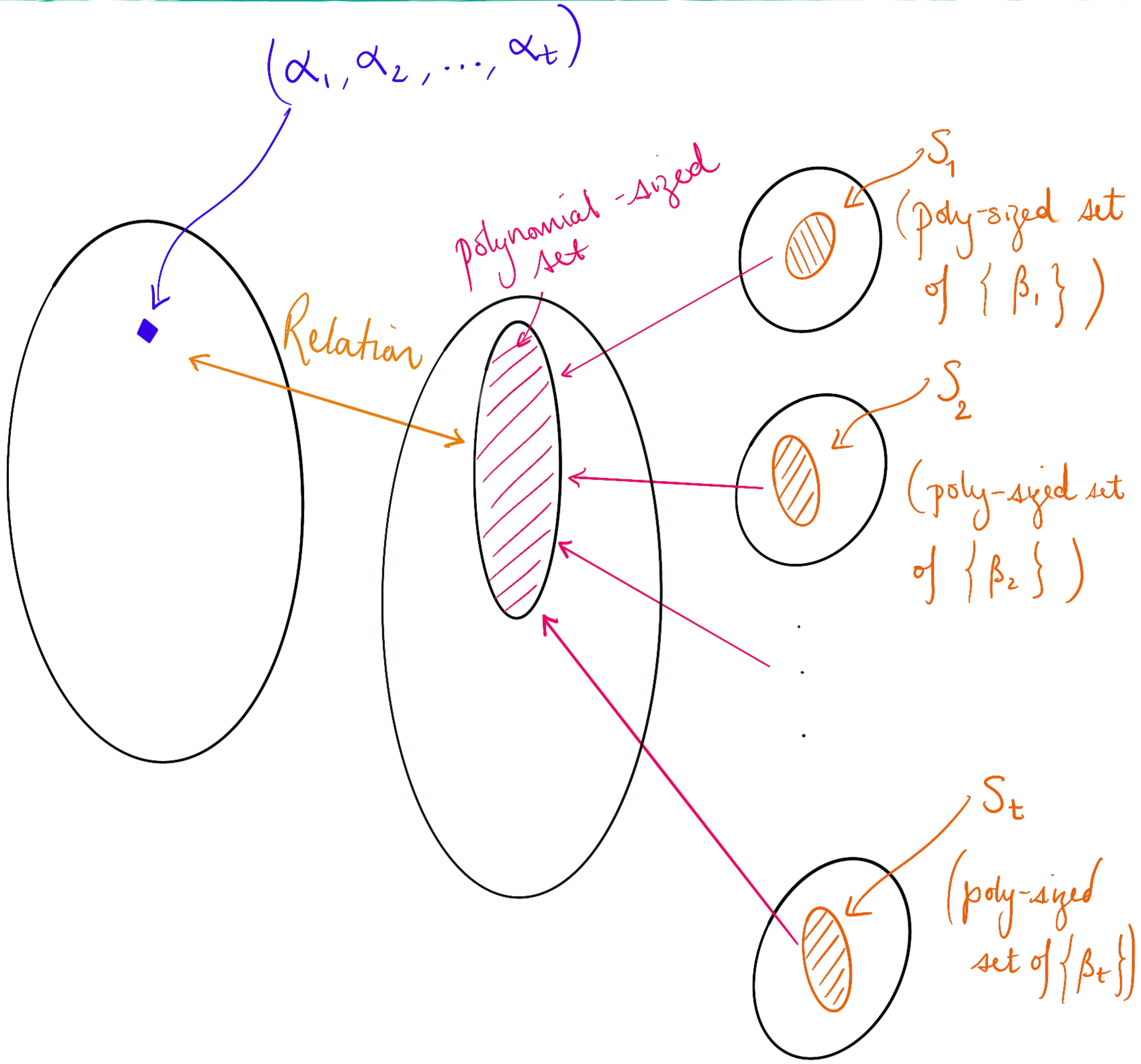

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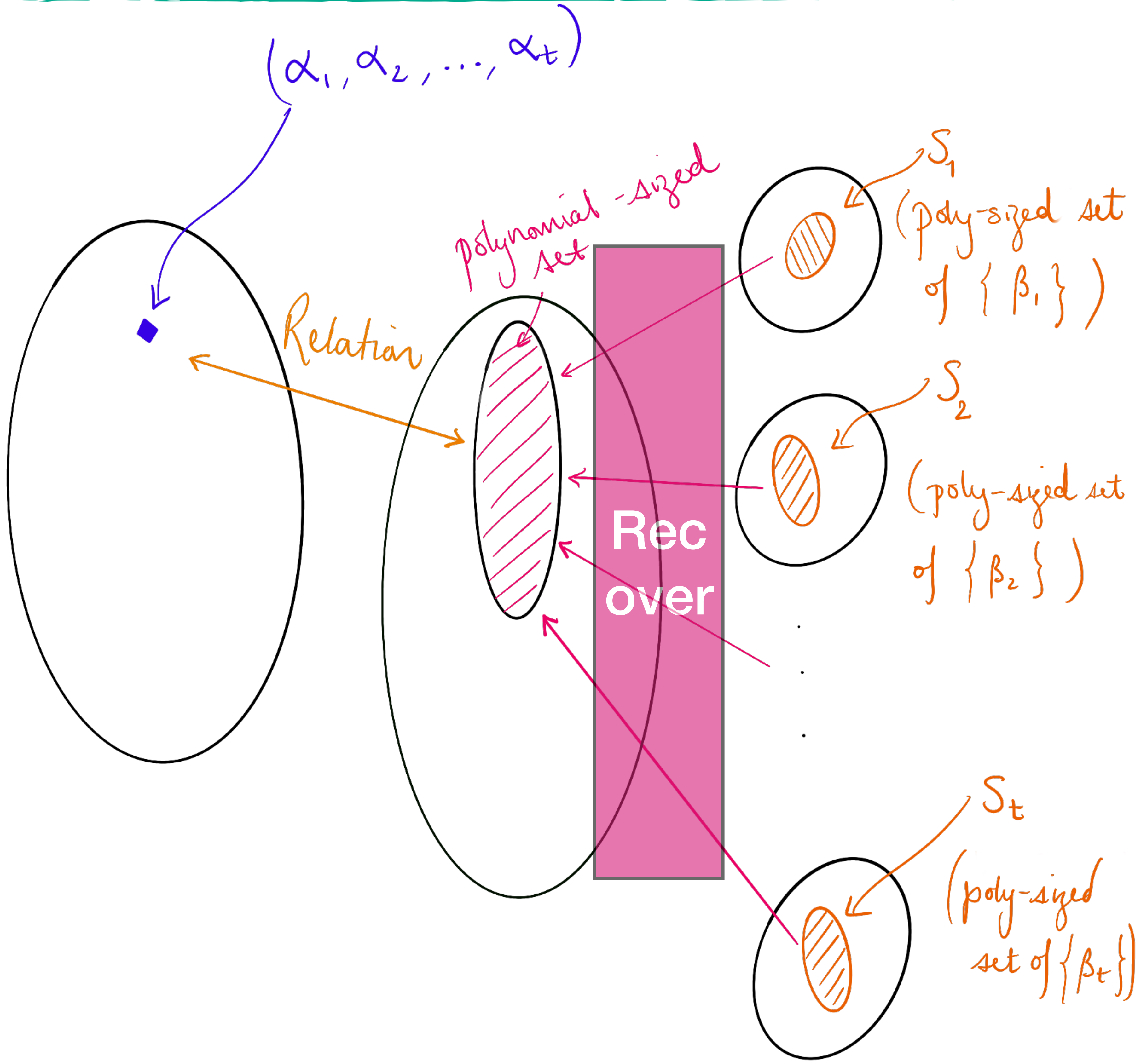
[HLR21] This is exactly list recovery!
Use a list-recoverable code!



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For a statement $x \notin L$:

$$R_x = \left\{ \big((\alpha_1, \ldots, \alpha_t), r\big) : (\text{Encode}(r))_i \in S_i \right\}$$

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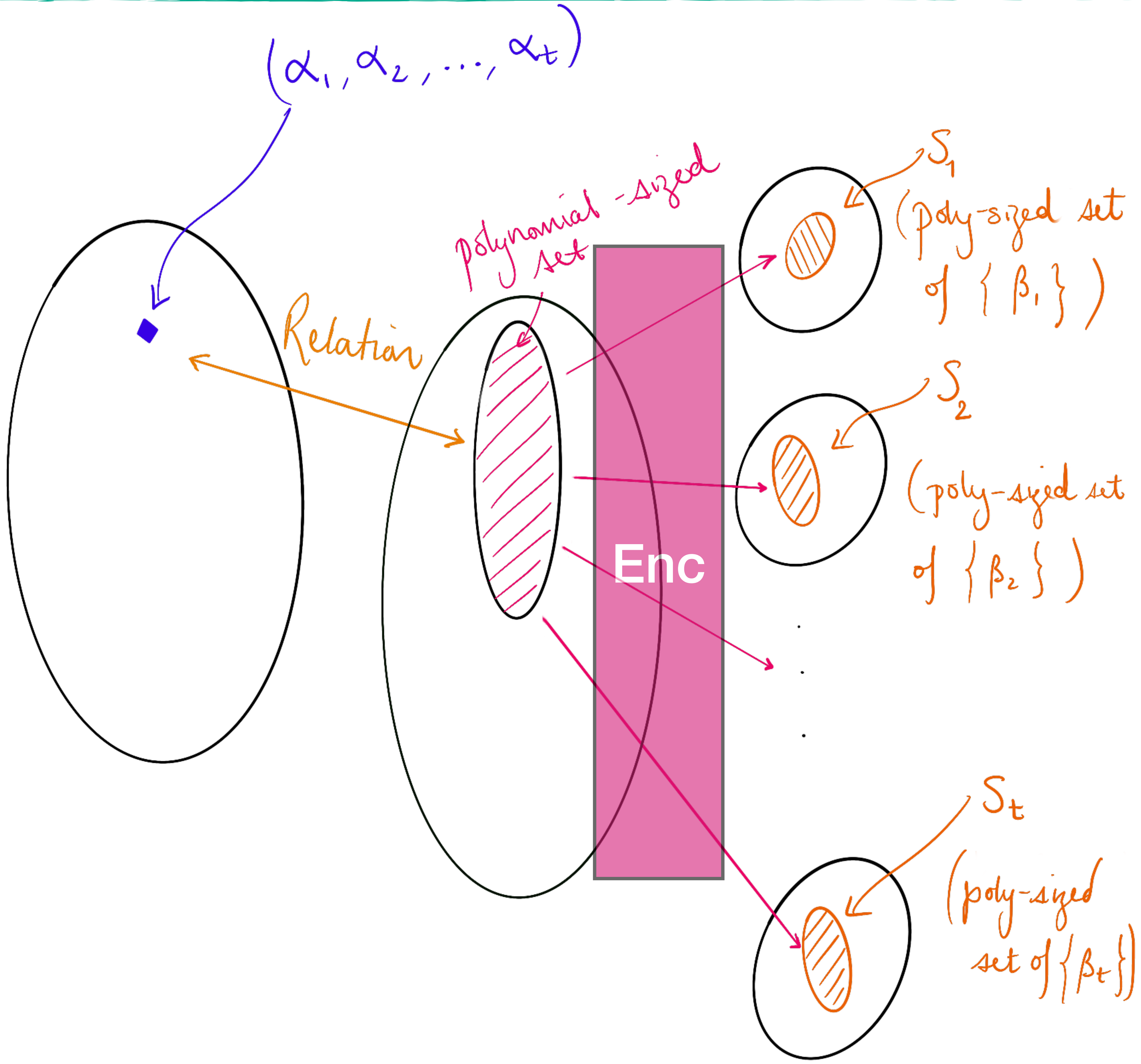

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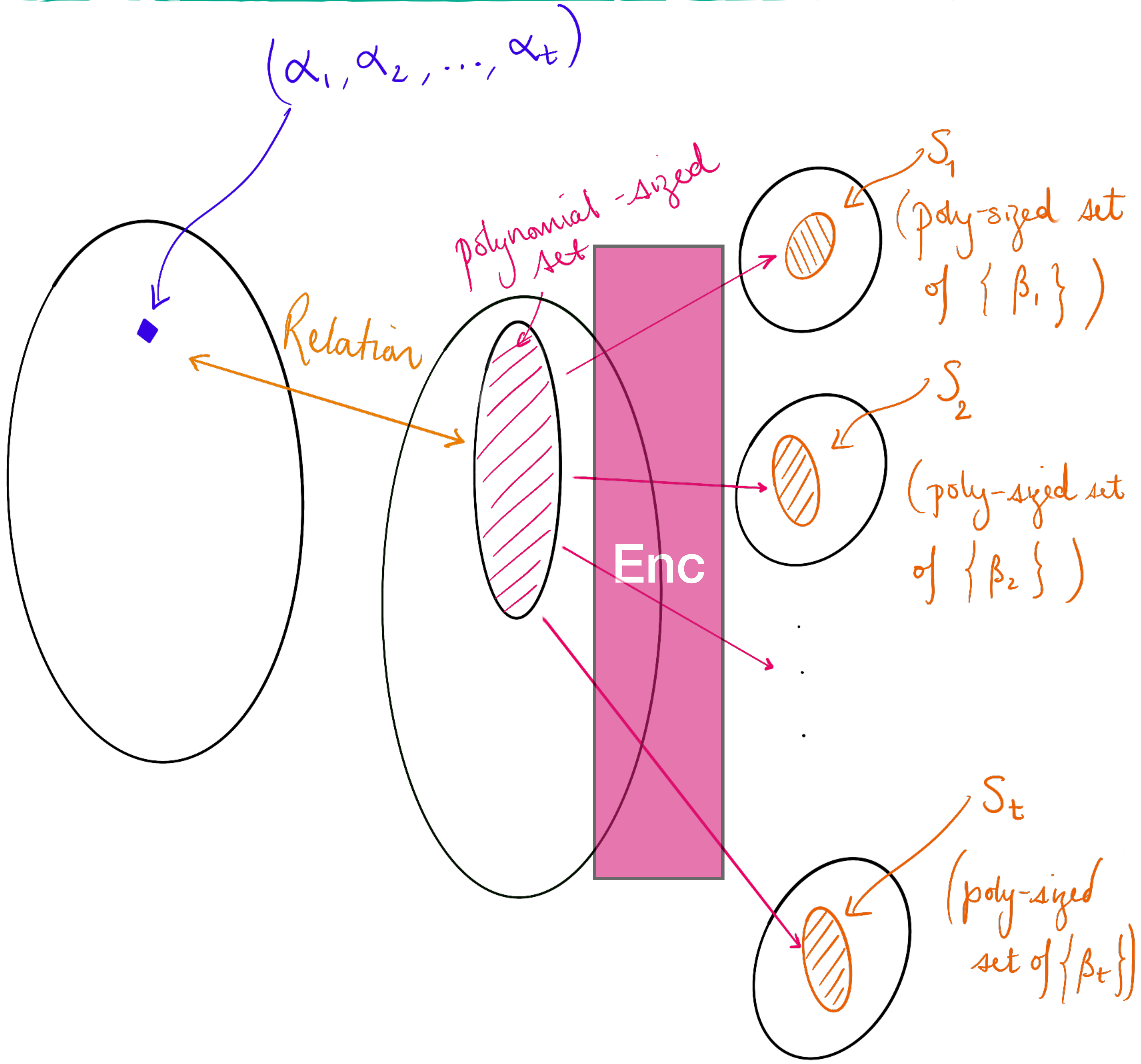
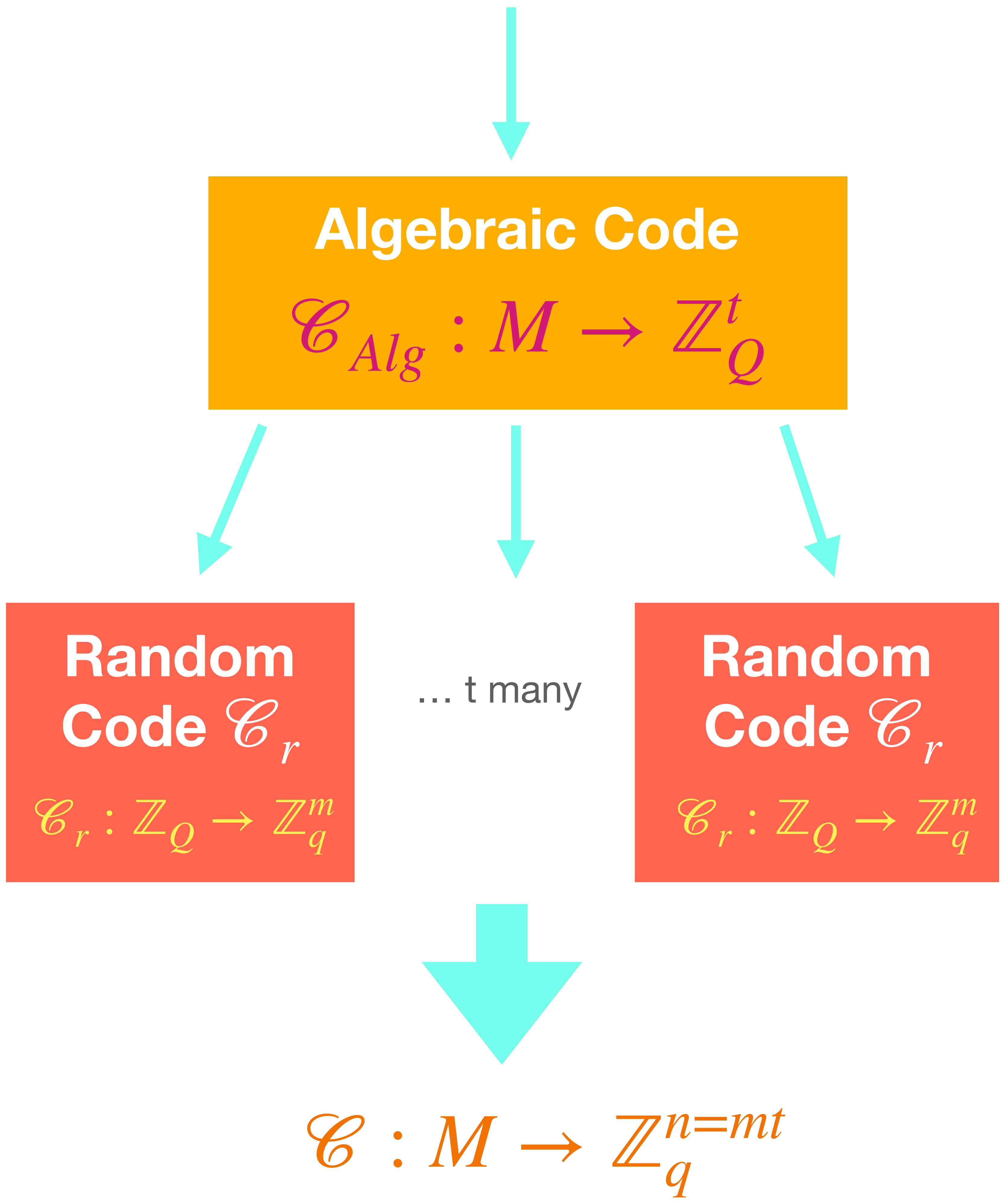
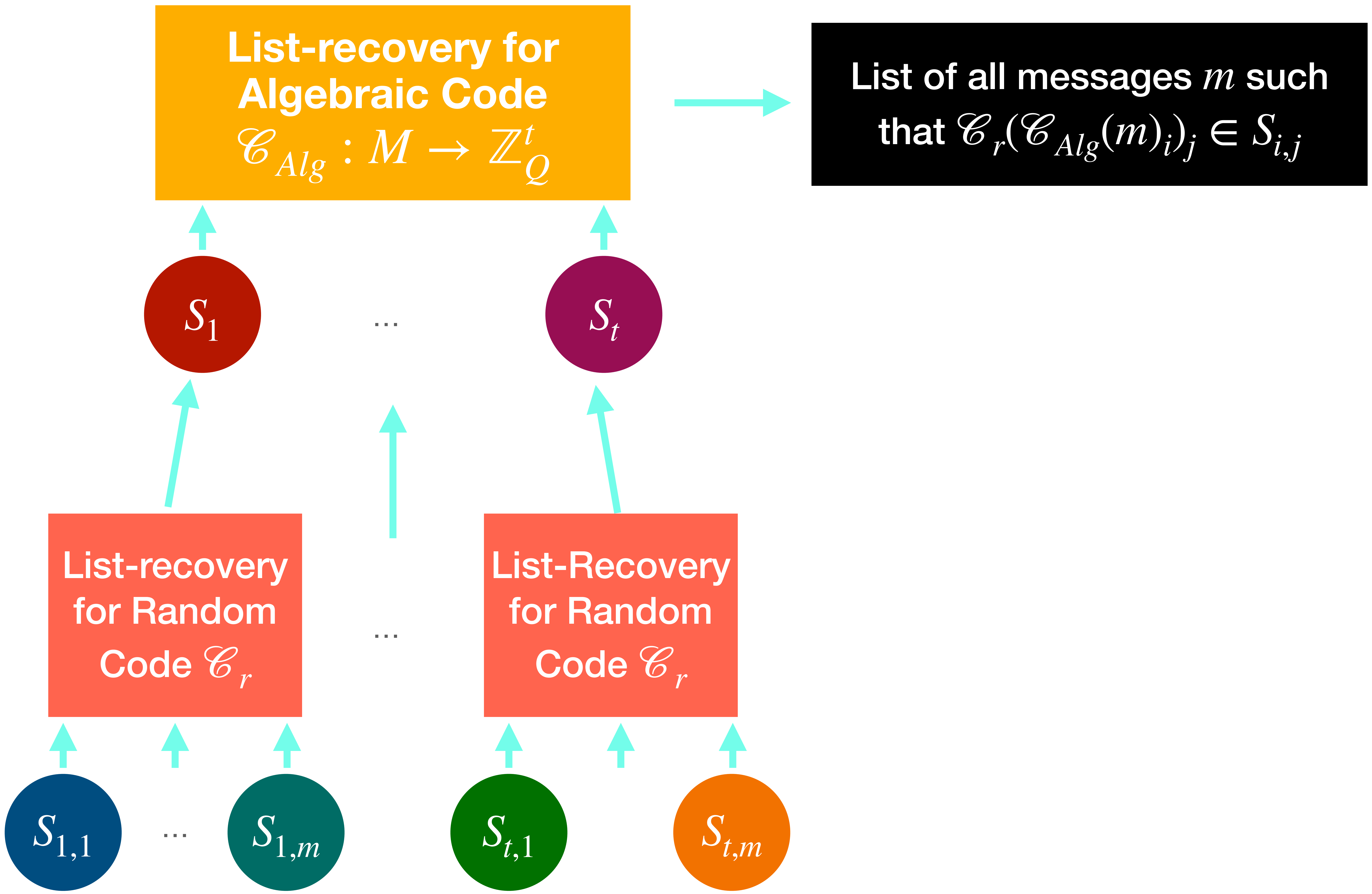
[HLR21] Use Parvaresh-Vardy code concatenated with a single random code.



Code Contenation



List-Recovery for Concatenated Codes



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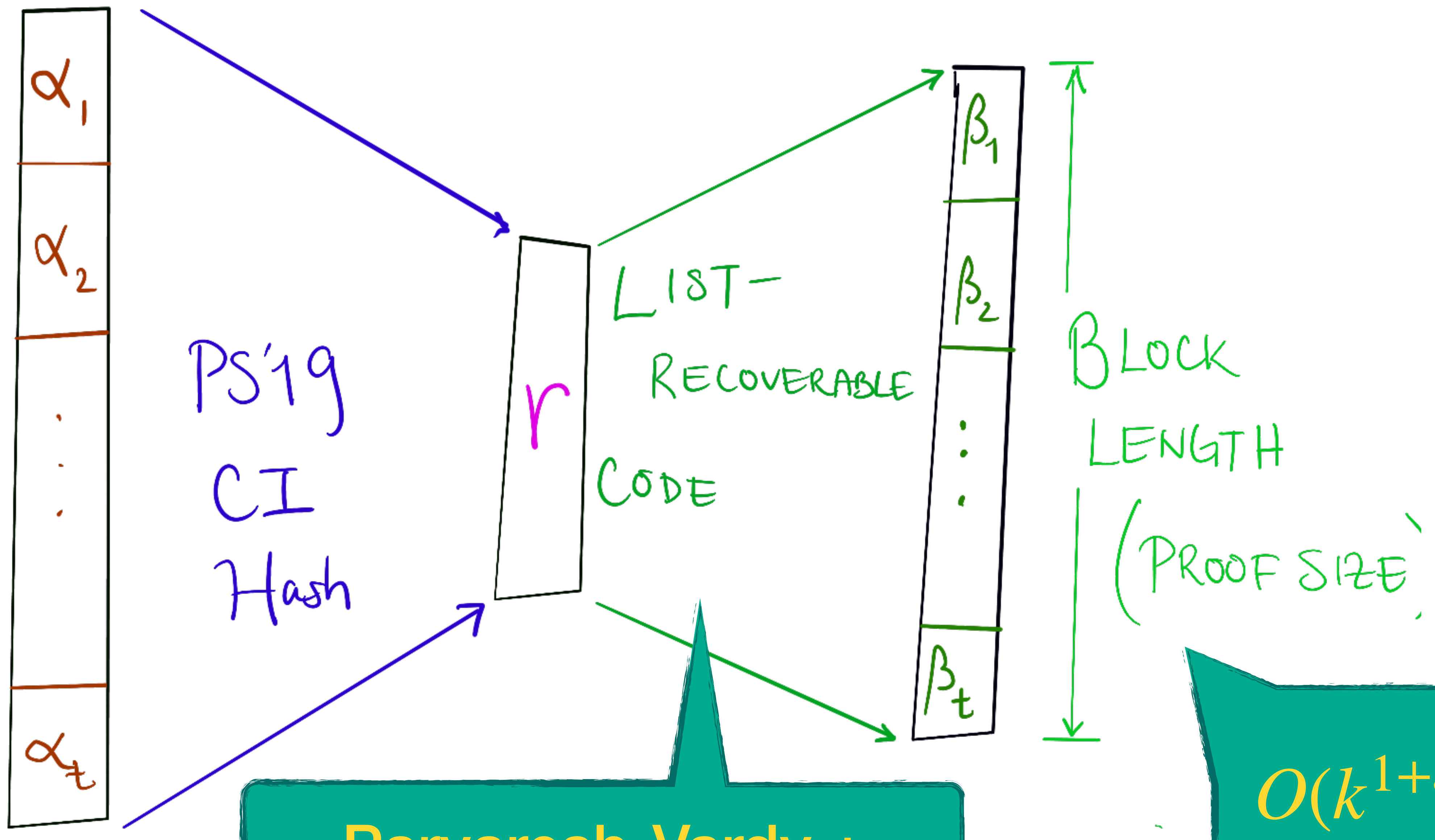
[HLR21] This is a CI hash for the desired relation.



Parvaresh-Vardy + Single Random Code

$O(k^{1+\varepsilon})$ for $\varepsilon > 0$

Fiat-Shamir from Coding Theory [HLR21]

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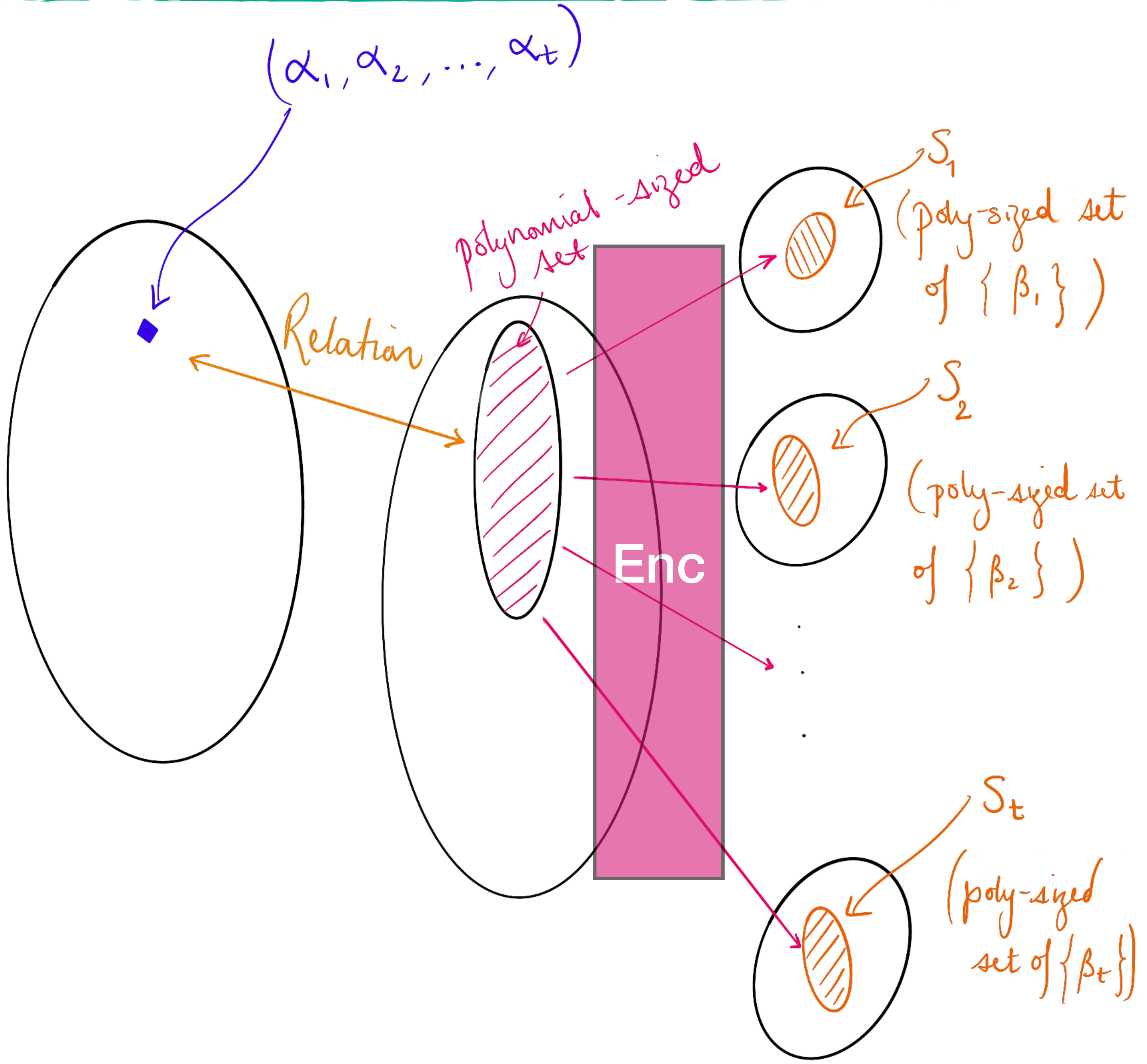
General list-recovery addresses product sets $S_1 \times S_2 \times \cdots \times S_t$ where each S_i may differ.



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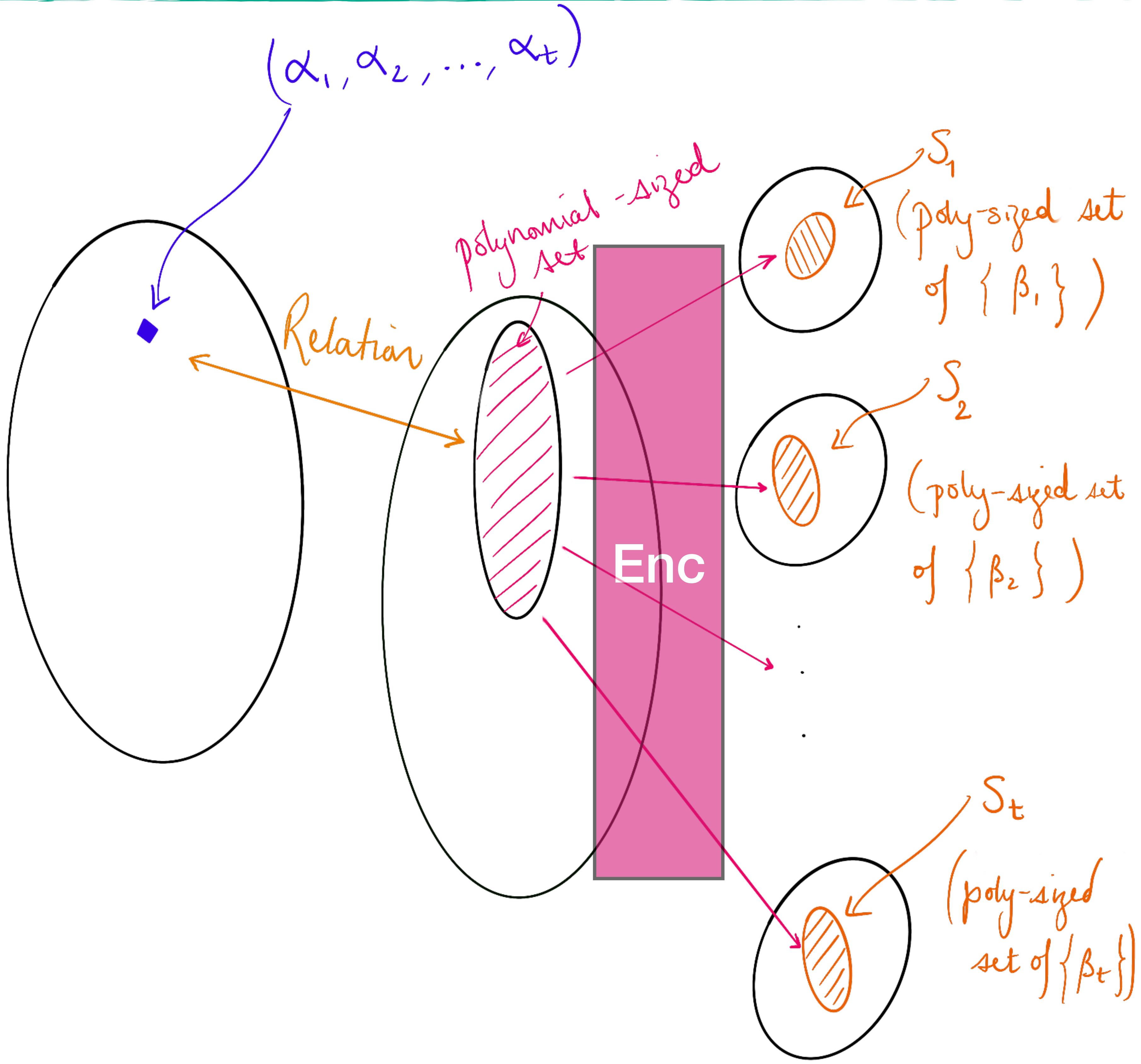
Is general list-recoverability necessary for the setting of MPC-in-the-Head?



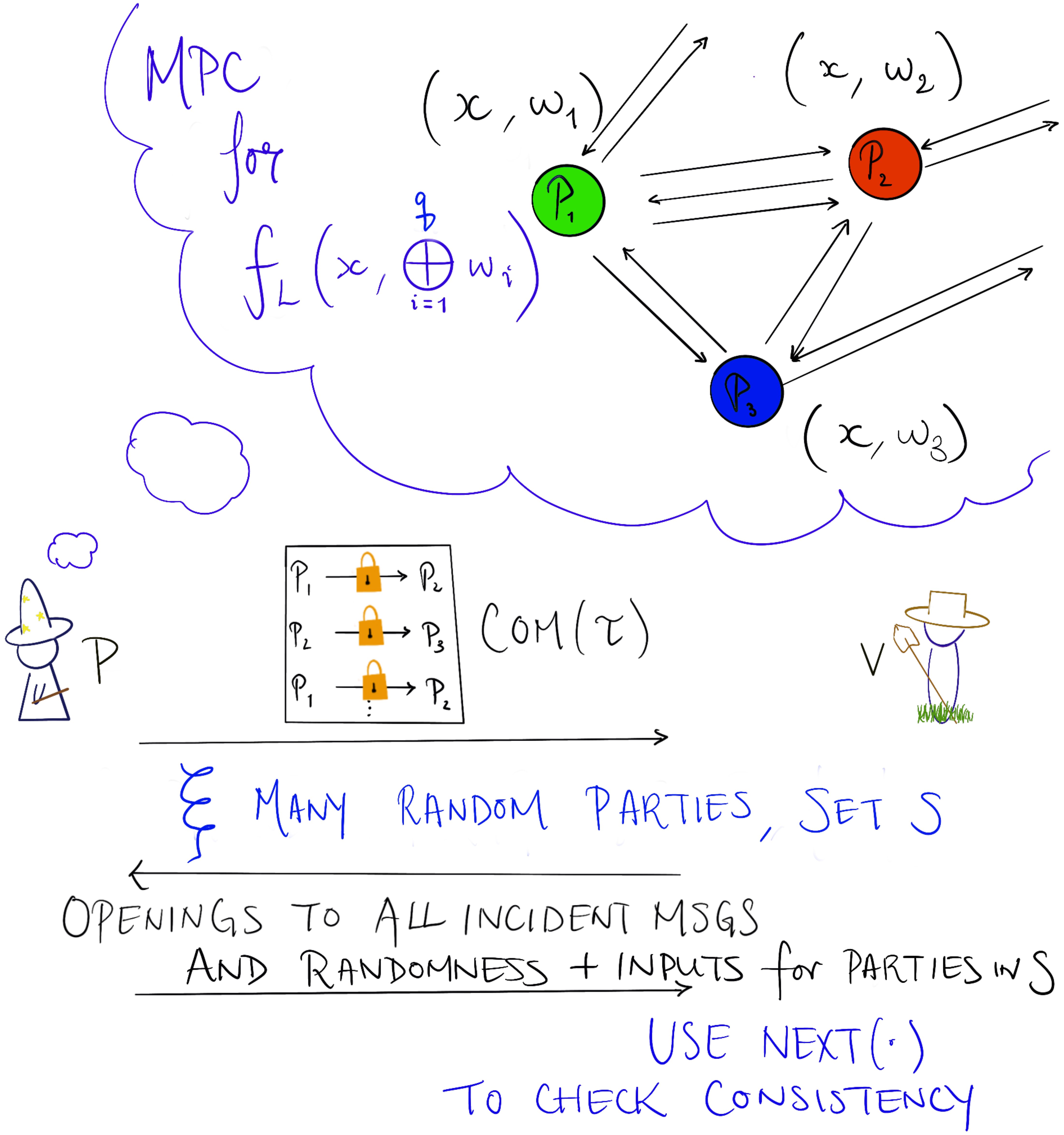
Bad Challenge Structure of MPC-in-the-Head

Bad Challenge Set:

$$S_{Com(\tau)} \times \cdots \times S_{Com(\tau)}$$

$$S_{Com(\tau)} = \{ i : \text{View}_i \text{ consistent } \} \subset \mathbb{Z}_q$$

For our MPC-in-the-head protocol, we have a product sets $S \times S \times \cdots \times S$ for a single set S, a much simpler structure.



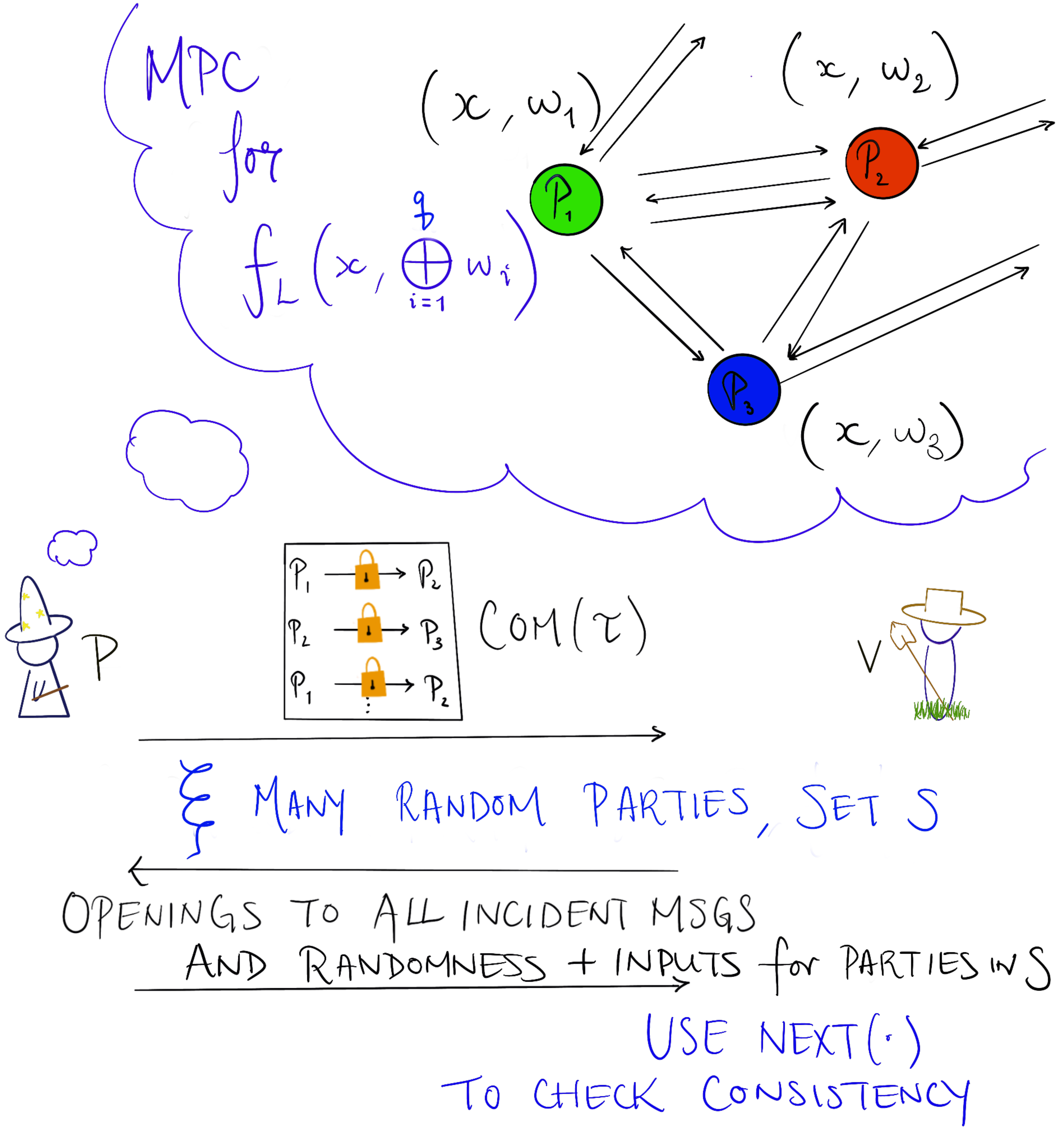
Bad Challenge Structure of MPC-in-the-Head

Bad Challenge Set:

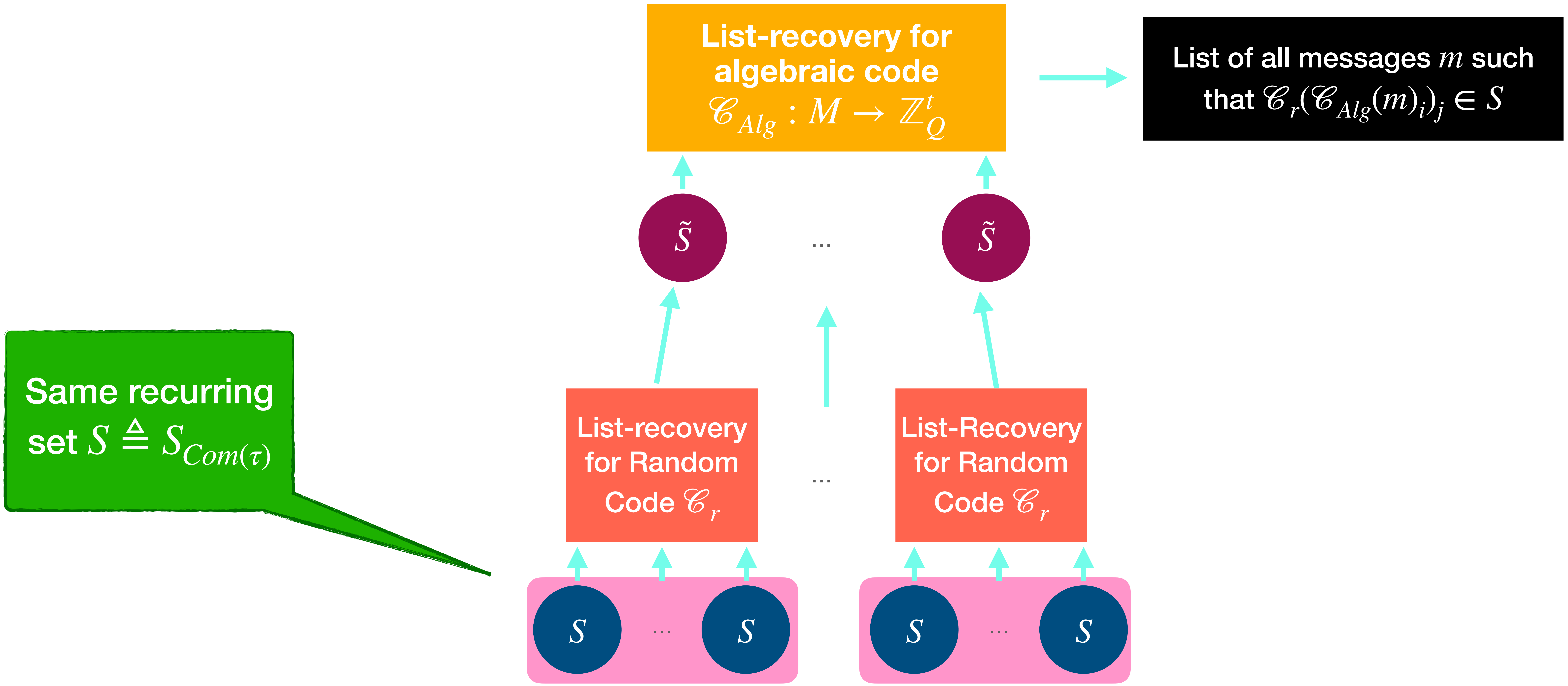
$$S_{Com(\tau)} \times \cdots \times S_{Com(\tau)}$$

$$S_{Com(\tau)} = \{ i : \text{View}_i \text{ consistent} \} \subset \mathbb{Z}_q$$

Does this simpler bad challenge structure allow the usage of a derandomization technique both *simpler* and *more efficient* than general list-recoverability?

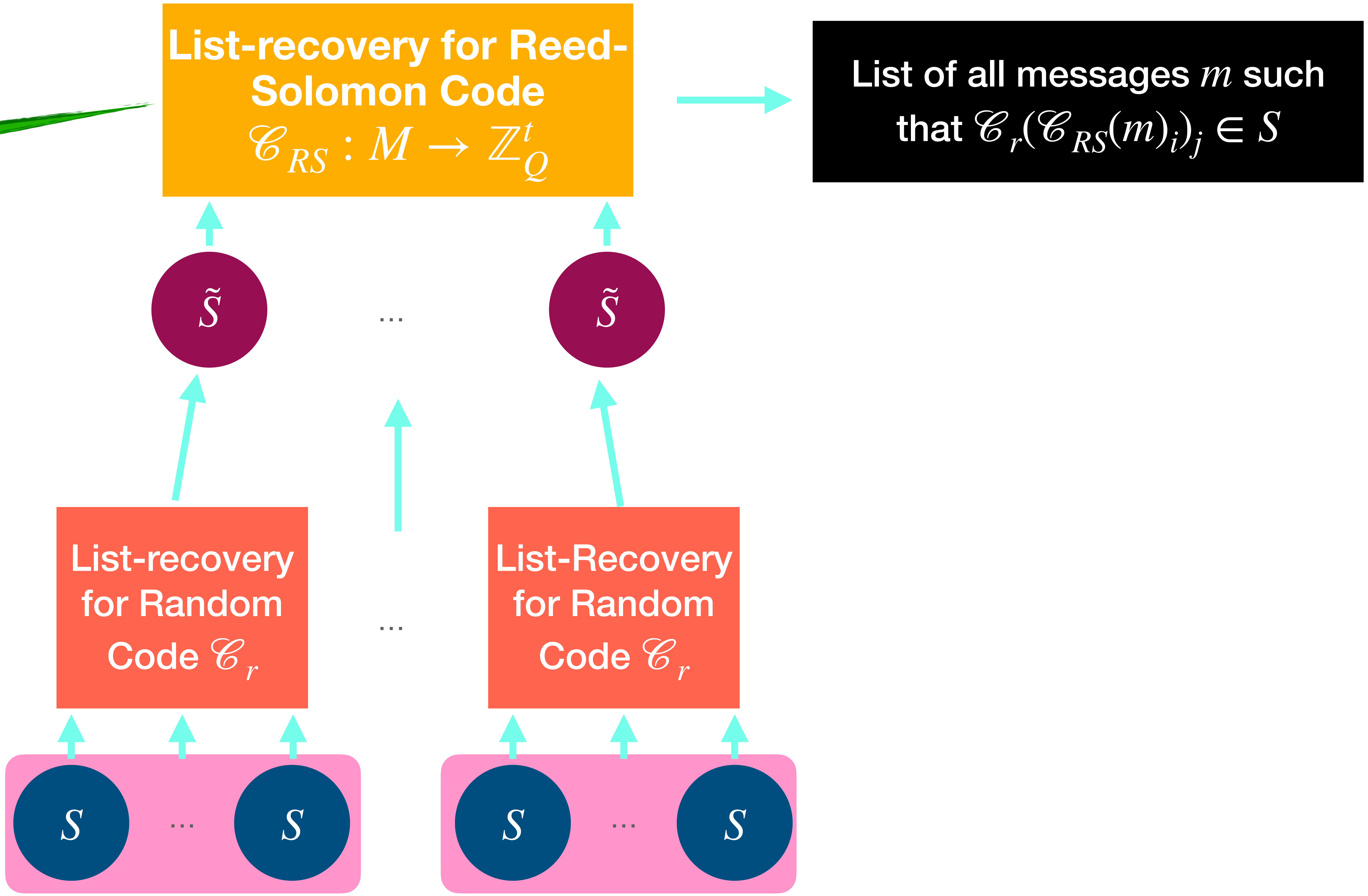


Recurrent List-Recovery



Recurrent List-Recovery

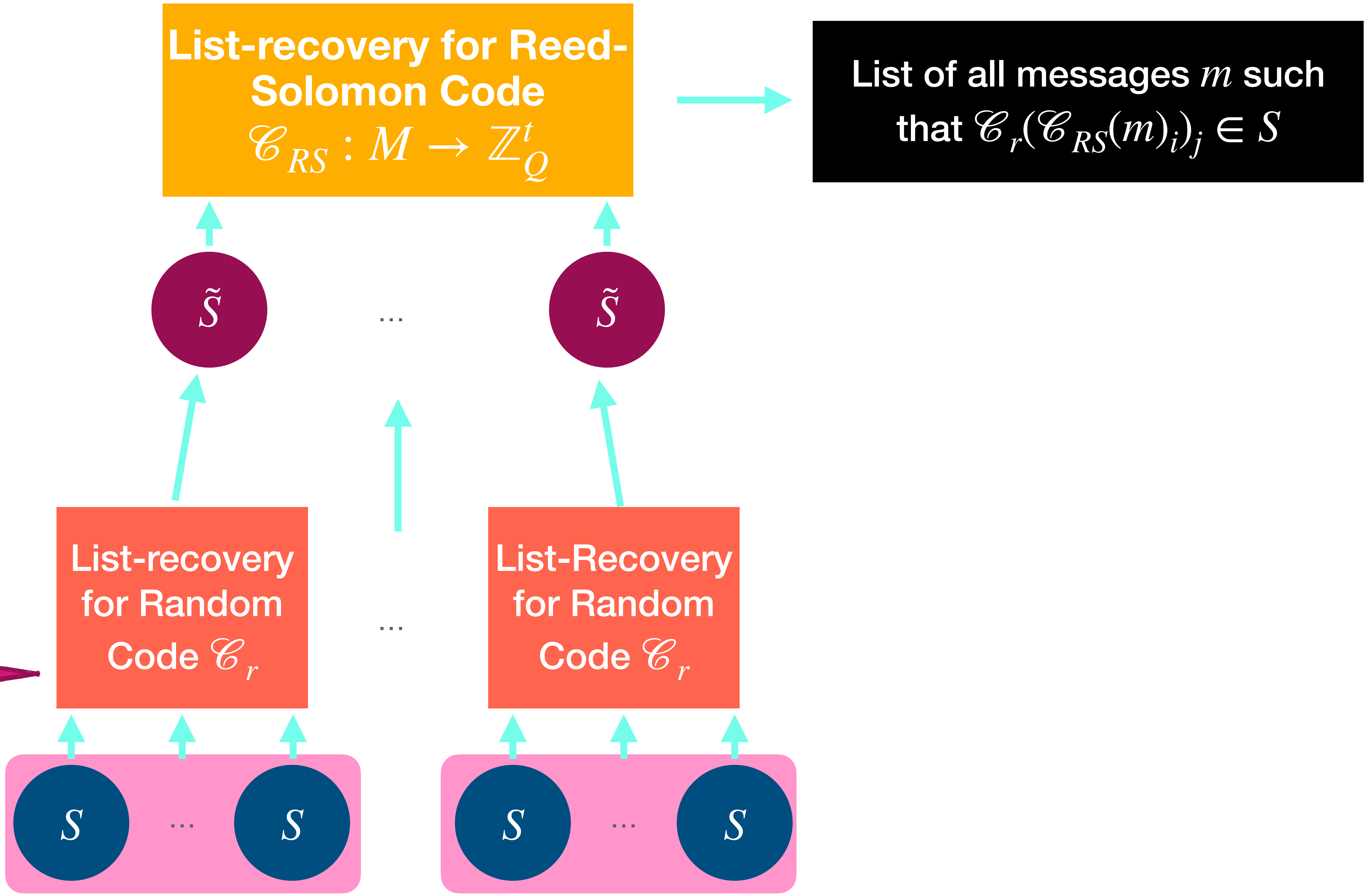
Let's try to use a simple algebraic code to instantiate recurrent list-recovery!



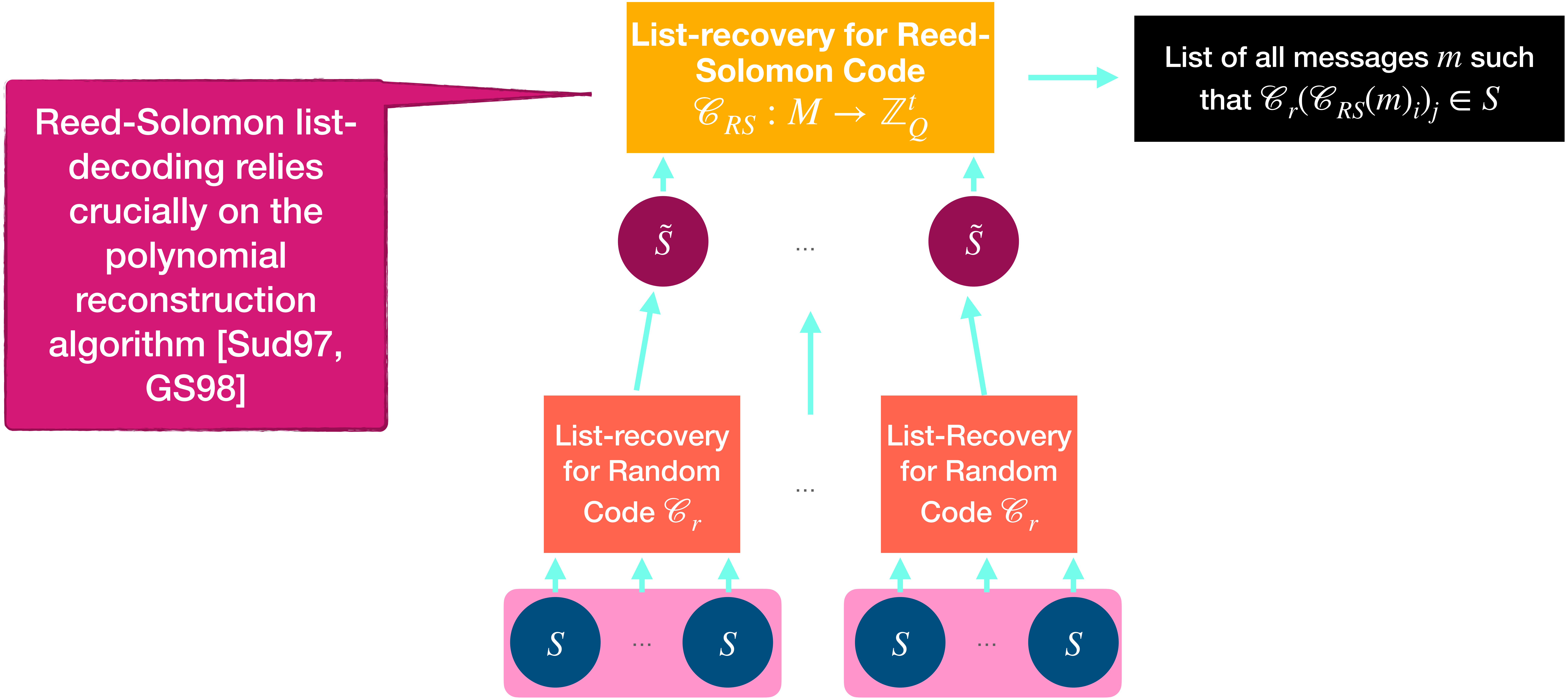
Recurrent List-Recovery

List-recovery for a *single* random code $\mathscr{C}_r$ may result in an output set $\tilde{S}$ that is too large for RS list-recovery!

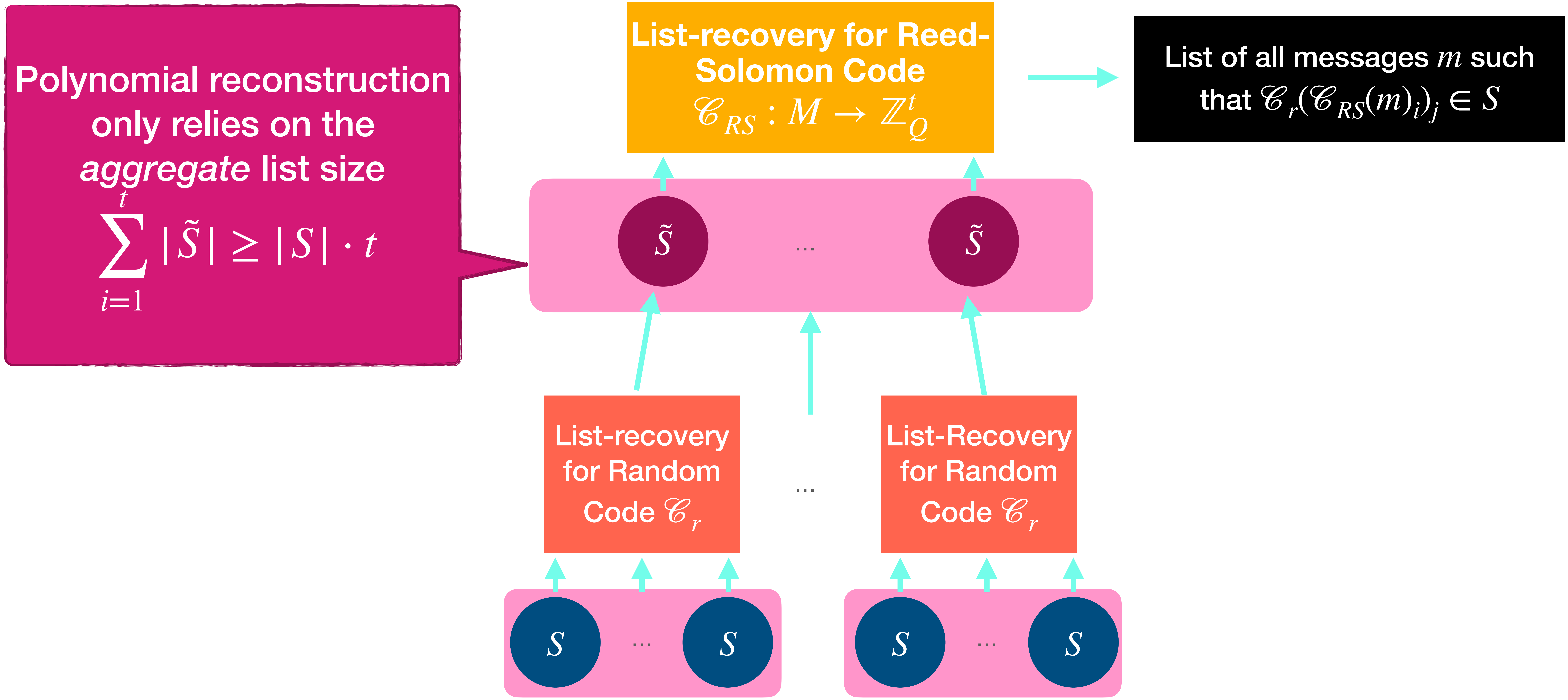
For a fixed random code, this happens with non-negligible probability over Prover's choice of S.



Recurrent List-Recovery

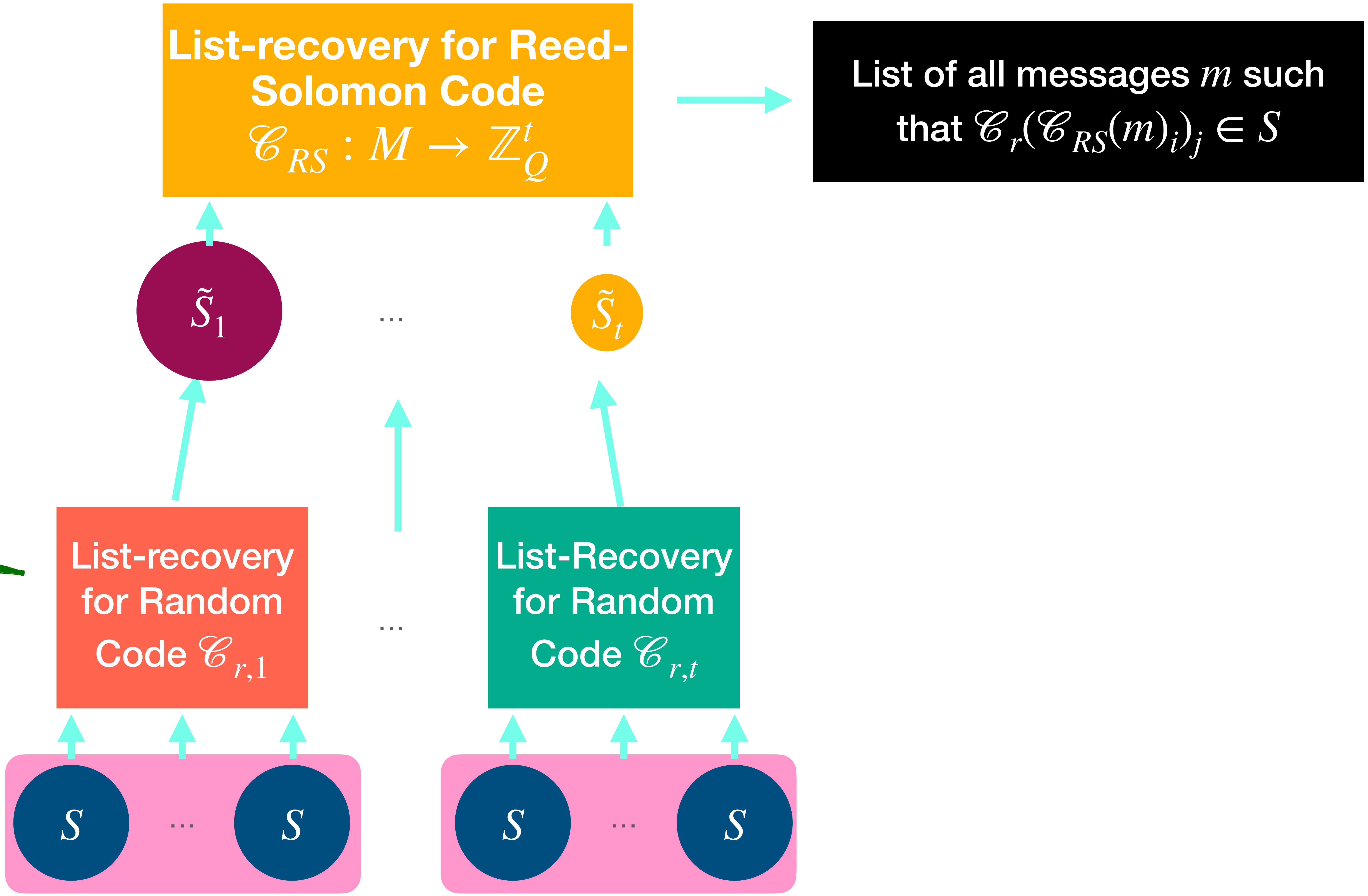


Recurrent List-Recovery



Aggregate Size Analysis

If we use *multiple* random codes, then while some output sets may be large, others may be small.

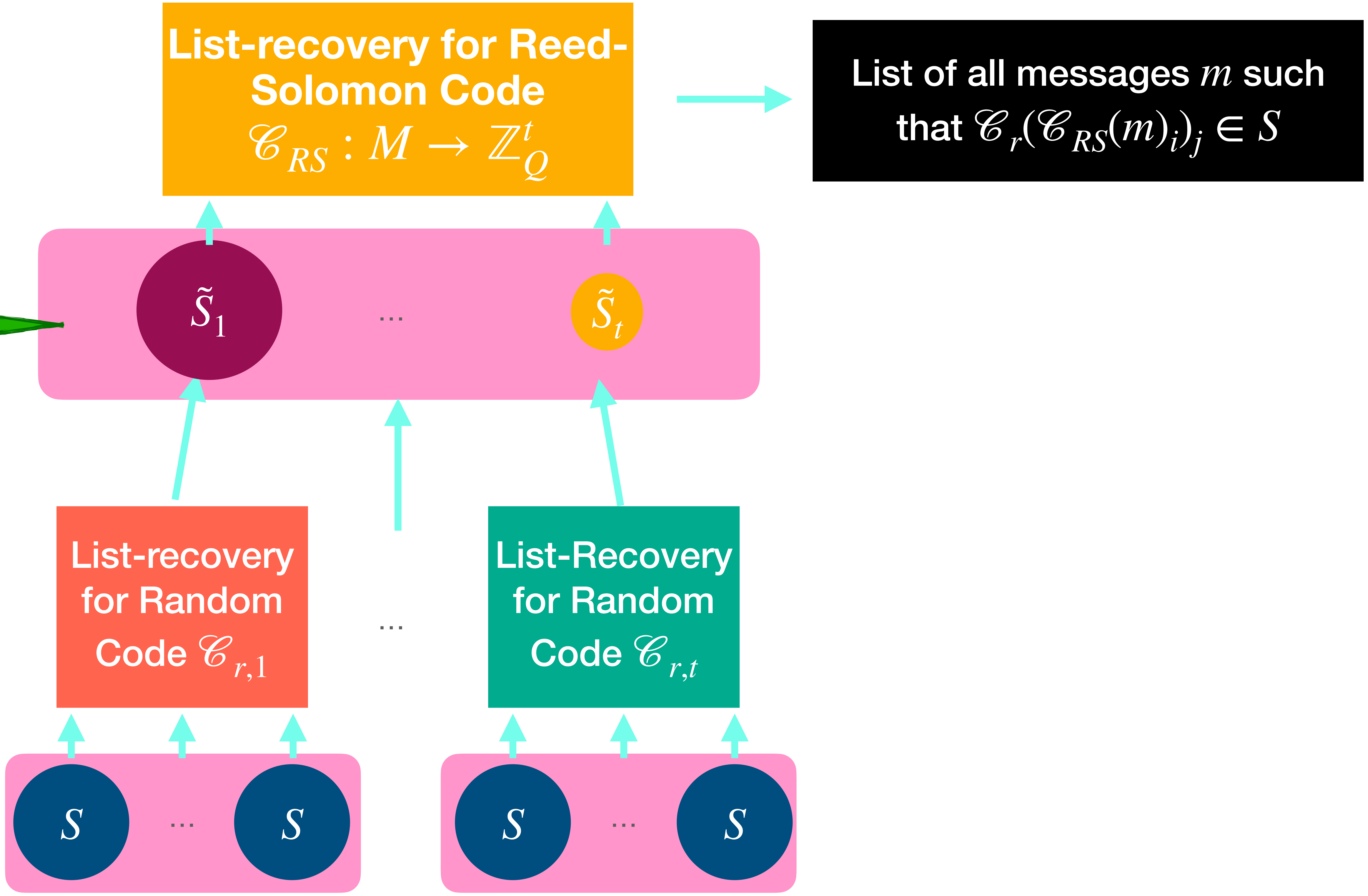


Aggregate Size Analysis

For $|S| = \alpha \cdot q$ for $\alpha \in (0,1)$, $q = \tilde{O}(k)$ we achieve

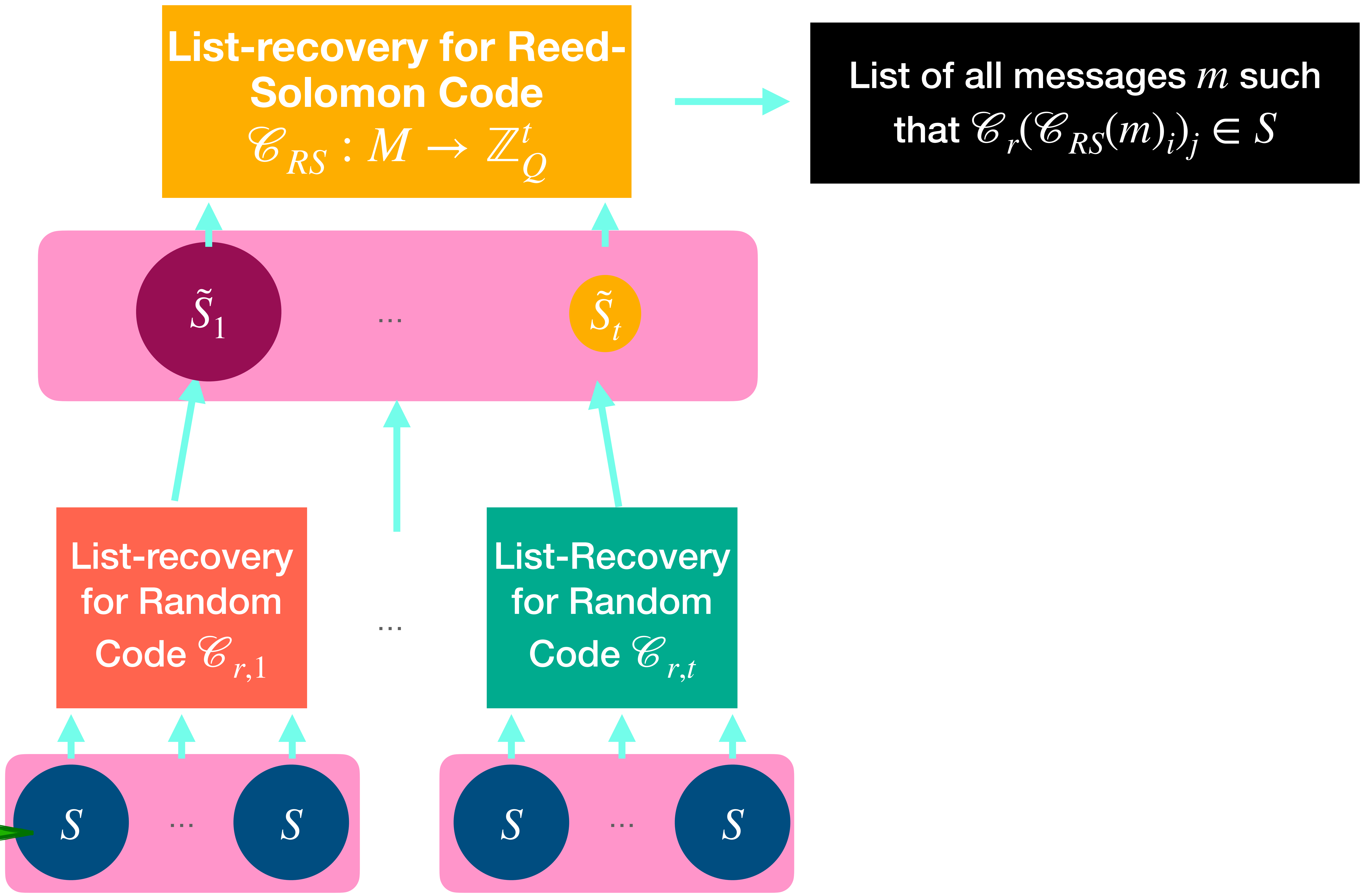
$$\sum |\tilde{S}_i| \leq \tilde{O}\left(|S| \right)$$

with all but negligible probability.



Aggregate Size Analysis

Polynomial reconstruction succeeds for every choice of the set S (of the appropriate size) with all but negligible probability.



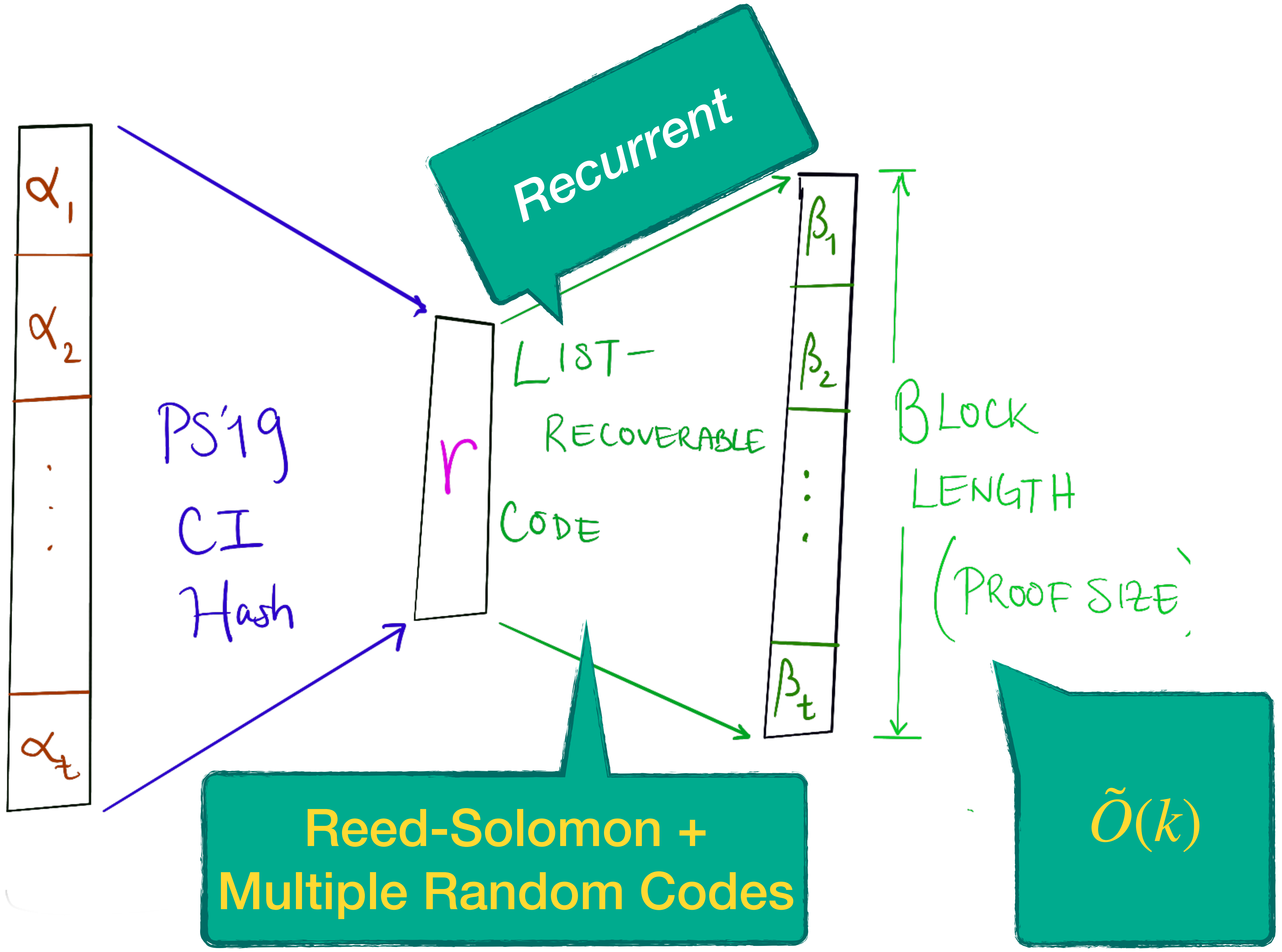
Summary:

We modify the MPC-in-the-head protocol [IKOS07] so that it has a bad challenge set amenable to ***recurrent list-recovery***. We instantiate the code with a Reed-Solomon code concatenated with multiple random codes, and use aggregate size analysis to obtain a quasi-linear block length!

For a statement $x \notin L$:

$$R_x = \left\{ \left((\alpha_1, \ldots, \alpha_t), (\beta_1, \ldots, \beta_t) \right) : \exists (\gamma_1 \ldots, \gamma_t) \text{ s.t. } V(x, \vec{\alpha}, \vec{\beta}, \vec{\gamma}) = 1 \right\}$$

This is still a CI hash for the desired relation.



Thank you!

Appendix

Reed-Solomon Codes + Polynomial Reconstruction

Def [RS60]: A Reed-Solomon code $\mathscr{C}_\lambda \colon \mathbb{Z}_Q^{k+1} \to \mathbb{Z}_Q^t$ is parameterized by a base field size $Q = Q(\lambda)$, a degree $k = k(\lambda)$, a block length $t = t(\lambda)$, and a set of values $A_\lambda = \{\alpha_1, \ldots, \alpha_t\}$. $\mathscr{C}_\lambda$ takes as input a polynomial p of degree k over $\mathbb{Z}_Q$, represented by its $k+1$ coefficients, and outputs the vector of evaluations $\big(p(\alpha_1), \ldots, p(\alpha_t)\big)$ of p on each of the points α_i.

Reed-Solomon Codes + Polynomial Reconstruction

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Polynomial Reconstruction:

- **INPUT:** Integers k_p, n_p. Distinct pairs $\{(\alpha_i, y_i)\}_{i \in [n_p]}$, where $\alpha_i, y_i \in \mathbb{Z}_Q$.
- **OUTPUT:** A list of all polynomials $p(X) \in \mathbb{Z}_Q[X]$ of degree at most k_p, which satisfy $p(\alpha_i) = y_i, \forall\ i \in [n_p]$.